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ABSTRACT

This volume is the seventh in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by kindergarten teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. The sections of this unit concern: (1) rotational symmetry, (2) repeating patterns, (3) bilateral symmetry; and (4) symmetry in sound and movement. A bibliography of books related to symmetry is provided for the teacher. (SD)

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# INTRODUCING SYMMETRY

ED127175

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

SE 021 212

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# INTRODUCING
# SYMMETRY

MINNEMAST COORDINATED MATHEMATICS-SCIENCE SERIES

UNIT 7

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

The Minnesota Mathematics and Science Teaching Project

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FIFTH PRINTING
1971

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

Director

DR. JAMES H. WERNTZ, JR.
Associate Professor of Physics
University of Minnesota

This unit, Introducing Symmetry, was developed from previous units and revised in the light of the experience of the many teachers who have tried the activities in the classroom. This trial edition, Unit 7 of the MINNEMAST Coordinated Mathematics-Science Series, was produced under the leadership of:

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CONTENTS


| | | |
|---|---|---|
| Materials List | | vi |
| Bibliography | | viii |
| Introduction | | 1 |
| Section 1. Rotational Symmetry | | 17 |
| Lesson 1 | Turning Blocks | 18 |
| Lesson 2 | Stars and S-Shapes | 20 |
| | Blue Transparency | 21 |
| | Yellow Transparency | 25 |
| Lesson 3 | Which Objects Have Turning Symmetry? | 30 |
| Art Activities for Section 1 | | 31 |
| Section 2. Repeating Patterns | | 39 |
| Lesson 4 | Paper-Doll Chain | 40 |
| Lesson 5 | Keep the Pattern | 41 |
| Lesson 6 | Find the Repeating Patterns | 43 |
| Art Activities for Section 2 | | 44 |
| Section 3. Bilateral Symmetry | | 48 |
| Lesson 7 | Faces and Hands | 53 |
| Lesson 8 | The Folding Test for Bilateral Symmetry | 56 |
| Lesson 9 | Mirror Test | 58 |
| Lesson 10 | Keep the Symmetry | 64 |
| Art Activities for Section 3 | | 68 |
| Section 4. Symmetry in Sound and Movement | | 76 |

Complete List of Materials for Unit 7

(Numbers based on class size of 30.)

(Art materials will depend on your selection of Art Activities.)

| total number required to teach listed lesson(s) | item | lessons in which item is used |
|---|---|---|
| 30 | *** sets of property blocks | 1 |
| 1 | * roll of masking tape | 1 |
| 1 each | ** blue and yellow transparencies | 2,4,8 |
| 1 | overhead projector | 2,4,8 |
| 12 | ** sheets of stars | 2,8 |
| 18 | ** sheets of S-shapes | 2,8 |
| 5-10 | assortment of objects, some rotationally symmetric | 3 |
| 30 | sheets of typing paper | Art, Section 1 |
| 40 | ** sheets of equilateral triangles | Art, Section 1 |
| | paste or glue | Art, Section 1 |
| 70 | sheets of construction paper | Art, Section 1 |
| 30 | thumbtacks | Art, Section 1 |
| 30 | pencils, with erasers | Art, Section 1 |
| 30 | pieces of tagboard | Art, Section 1 |
| | scraps of colored paper or yarn | Art, Section 1 |
| 10 or 30 | sets of objects from which patterns can be made, e.g., property blocks | 5,10 |
| 5-10 | assortment of objects and designs, some exhibiting repeating patterns | 6 |

| | | |
|---|---|---|
| 100 | sheets of construction paper | Art, Section 2 |
| | tempera paints and brushes | Art, Section 2 |
| | paste | Art, Section 2 |
| | small, flat objects | Art, Section 2 |
| 35 each | **Design Sheets 1 and 2 | 9 |
| 30 | *small mirrors | 9 |
| 1 | large mirror | 9,10 |
| 30-40 | assortment of small, solid objects | 9 |
| 1 | flannel board and sets of objects | 10 |
| 35 | **Design Sheet 3 | 10 |
| 30 | sheets of rice paper or newsprint | Art, Section 3 |
| 60 | sheets of construction paper, pencils and peg board | Art, Section 3 |
| 30 | sheets of newsprint, crayons or soft charcoal, and objects with raised patterns | Art, Section 3 |
| 30 | sheets of 9" x 12" construction paper | Art, Section 3 |
| 30 | sheets of 12" x 18" construction paper | Art, Section 3 |
| 30 | sheets of construction paper, tempera paint, egg cartons, and *pipe cleaners | Art, Section 3 |
| 60 | sheets of construction paper and paste | Art, Section 3 |

*kit items as well as

**printed materials available from
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Arts

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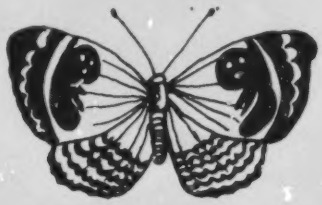
## INTRODUCTION

In popular usage the word symmetry often carries the implication that a thing is pleasantly proportioned or well-balanced. To many, symmetry implies a type of beauty and perfection. In precise usage, a symmetric pattern is one which remains unchanged under a certain rigid motion. One can intuitively notice symmetry in many things such as:

butterflies
leaves
tile floors
stanzas of poetry
snowflakes
the arrangement of seeds in a sunflower
melodies
rhythms
the weaving in straw baskets
slices of citrus fruits
mathematical principles

Certain types of symmetry can be given precise geometric descriptions. Three of these types are emphasized in this unit: (1) rotational or turning, (2) translational and repetitive, (3) bilateral or symmetric about a line. A detailed discussion of them will conclude this commentary. It will be found that symmetric patterns are unchanged by certain motions.

You need not feel that a complete mastery of the introductory material is necessary before you start to teach the unit. Each idea is demonstrated and developed as the lessons progress. This introduction should be regarded as a source of information to which to refer for information during the various studies.



## Why Have Symmetry Units in the Primary Grades?

Both the intuitive ideas of symmetry and the precisely defined mathematical ideas of symmetry were considered in the following reasons for having small children study symmetry.

1. The activities provide practice in abstracting concepts, e.g., seeing that several different patterns have the same type of symmetry.

2. The concepts discussed will be used in later topics such as geometry, measurement, wave motion, and classification of biological organisms. Symmetry is a tool in understanding the universe.

3. The principles of symmetry are often applied in art, poetry, and music. Awareness of these principles heightens appreciation of the arts.

4. The activities provide perceptual training for reading readiness. For example, the tracing of patterns aids perception through muscular activities. Practice is given in moving from left to right.

5. The children enjoy the activities and the awareness of symmetry that they develop. Many natural and man-made shapes in the children's environment have some type of symmetry that they can recognize.

## Objectives

On completion of the unit the child should:

1. Recognize symmetry in nature, music, poetry, and art.

2. Recognize turning symmetry (rotational symmetry), repeating patterns (translational symmetry) and symmetry about a line (bilateral symmetry).

3. Be able to make the necessary tests and comparisons for the three types of symmetry studied.

## Rotational Symmetry or Turning Symmetry

Consider the letter S. Think of it as a shape that can be moved and which leaves its image behind. Can the S be moved in a definite way so that after it has been moved it will coincide with its original image?

1. Simply picking the S-shape up and laying it back down is one such operation. This is trivial because it can be done for any movable object.

2. Turning the S-shape a full turn, i.e., through $360^{\circ}$, is another move that will leave the shape in coincidence with its original image. This is also trivial because it can be done for any shape.

3. Turning the S-shape through one half of a full turn, i.e., through $180^{\circ}$, will also cause the S to coincide with its original shape.

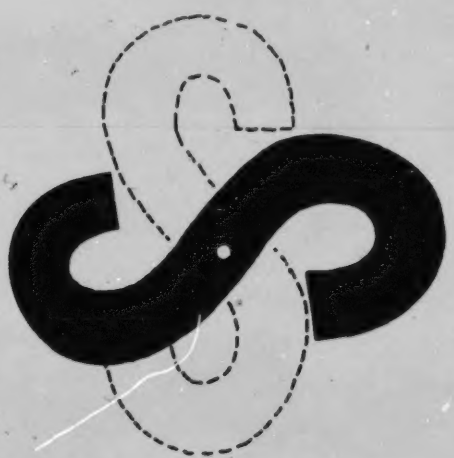


Figure 1

This third operation shows that the S-shape is symmetric with respect to rotation about an axis perpendicular to the shape at the center. The shape is said to have rotational symmetry or turning symmetry. Note that if the S is flipped over to the position Ƨ it cannot be put into coincidence with its original shape.

Another example of a shape that possesses rotational symmetry is a triangular block with equal sides. The block outlines in Figure 2 may help in visualizing the rotations that leave the block in coincidence with its original image. The rotations are one-third of a full turn, two-thirds of a full turn, and the trivial ones of no turn and of a full turn; rotations of $0^{\circ}$, $120^{\circ}$, $240^{\circ}$ and $360^{\circ}$ leave the pattern of the block unchanged.

Figure 2

Note that in considering rotational symmetry of a pattern, only rotations about an axis through the center of the pattern are considered.

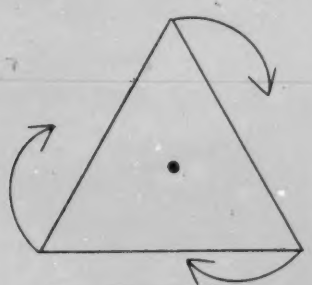


Figure 3

The pattern is not flipped or turned about an axis across the pattern.

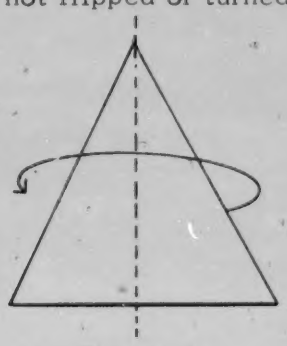
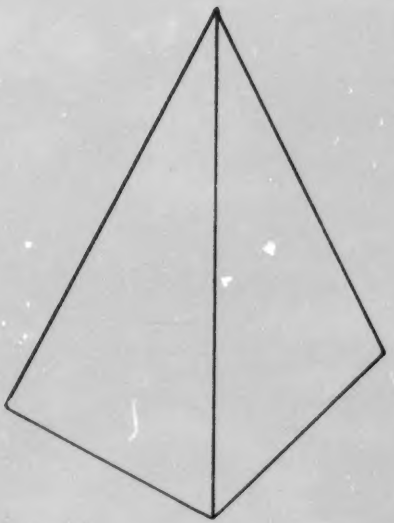
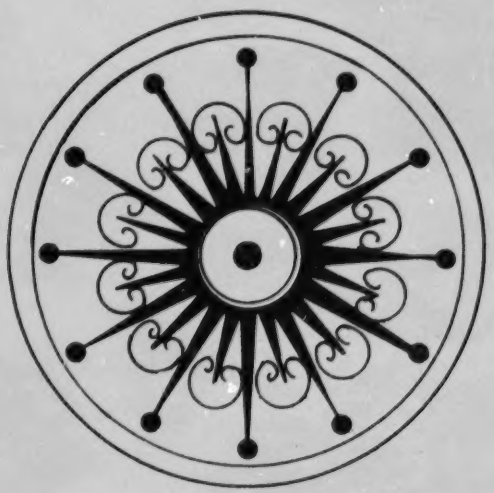


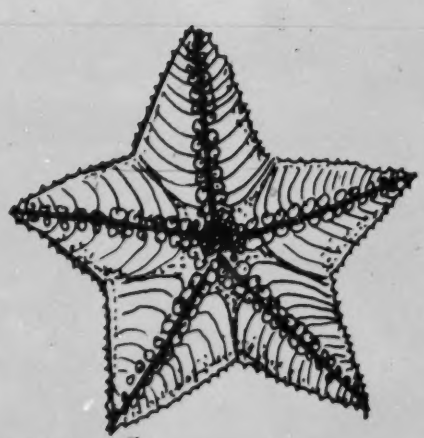
Figure 4



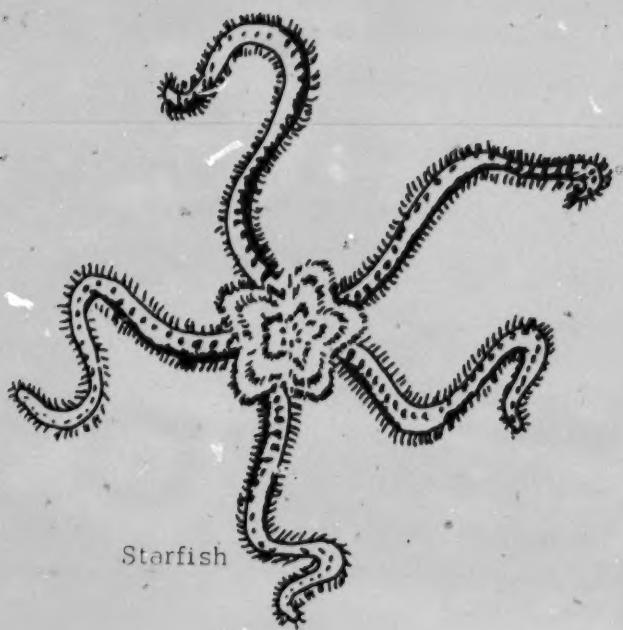
Pyramid



Plate



Starfish



Starfish

Figure 5
EXAMPLES OF ROTATIONAL SYMMETRY

A rotation through any amount less than a full turn that yields coincidence with the original position is a test for the presence in a pattern of rotational symmetry or turning symmetry. Further examples of simple patterns exhibiting rotational symmetry are stars, squares, ovals, and circles.

A three-dimensional object can't be tested for rotational symmetry exactly as done above, because a solid object won't coincide with a surface pattern. However a variation of the test can easily be made. The solid object can be turned and compared to an exact copy of itself. If, after the object has been turned an amount less than 360°, the object and its copy cannot be distinguished, the object has rotational symmetry. Figure 5 shows a pyramid and a plate, each with exact rotational symmetry, and two starfish showing approximate rotational symmetry.

A slightly different point of view is to consider a rotationally symmetric pattern as one that can be generated by rotating an element of the pattern. For example, the shape can be generated by rotating the shape into three positions.

Repeating Patterns and Translational Symmetry

Suppose a stencil is used to make the repeating pattern in the strip design shown in Figure 6.

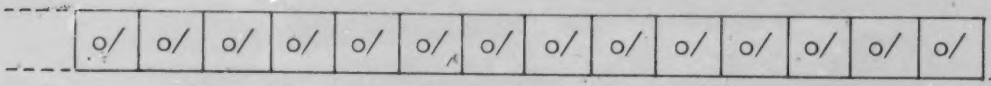


Figure 6

The dotted lines in the figure show that the pattern could be continued indefinitely. If the pattern is imagined as being infinitely long, it can be picked up, moved through a certain distance, and laid down again to cover the entire original pattern. The pattern would then look exactly as it did before it was moved. This is possible because an infinite pattern has no beginning and no end, but extends infinitely in either direction. A mathematician might say that a translation carries the pattern into itself. Because of this, the pattern is said to possess translational symmetry.

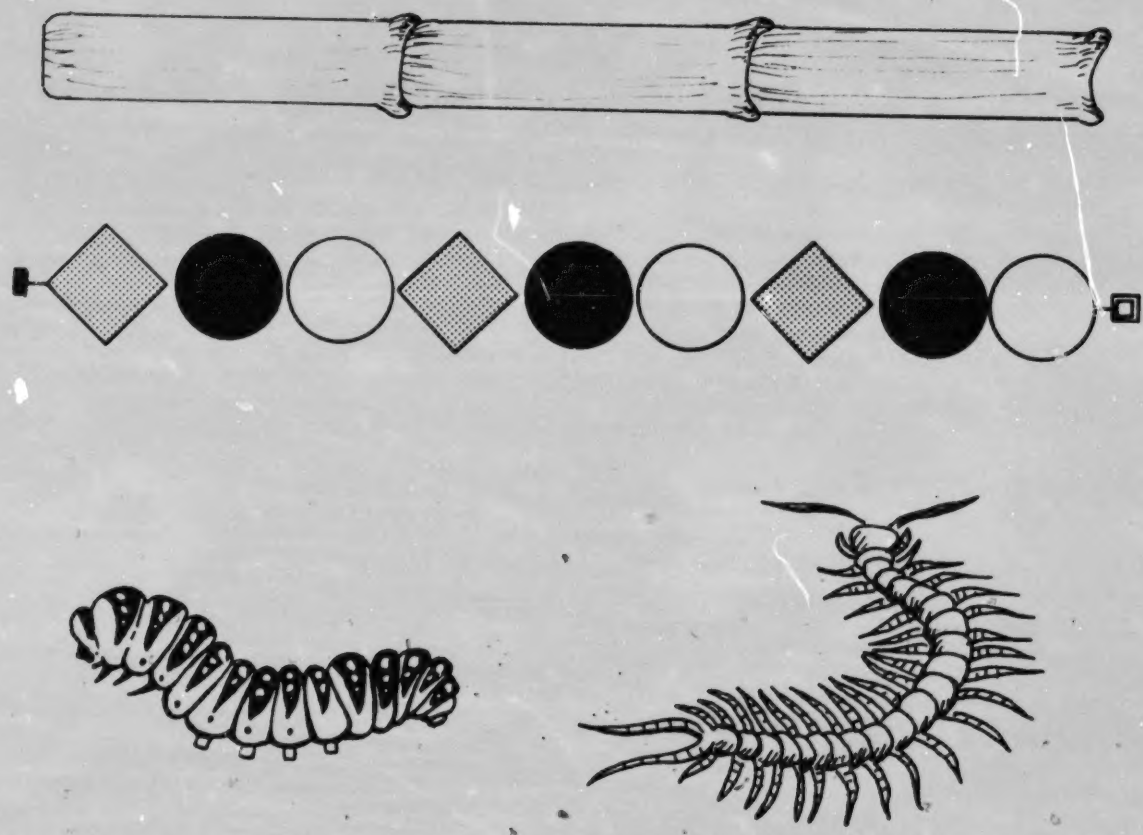
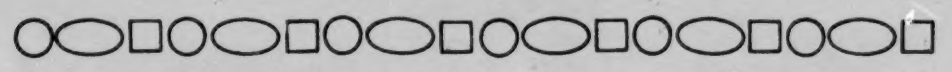


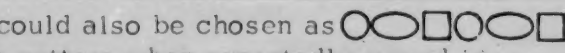
Figure 7
EXAMPLES OF REPEATED PATTERNS

Repeating patterns are easy to find. For example, they are seen in wallpaper, brick walls, and strings of beads. These patterns repeat only a finite number of times, but they may be thought of as "potentially infinite." This means that the pattern can be imagined to extend indefinitely in both directions. Thus the term "translational symmetry" is sometimes used in talking about repeating patterns which are finite but can be imagined to extend infinitely.

A pattern is a repeating pattern if some part of the pattern (1) can be moved a definite distance along a line to cover an identical part of the same pattern and (2) with a succession of such moves will cover the entire pattern. As in rotational symmetry, comparisons are made after some part of the pattern is moved. This time a part of the pattern is moved in equal-sized jumps and compared to other parts of the same pattern to determine coincidence. The part of the pattern that is repeated can be called a "block," a "cell," or an "element."

In testing the string of beads shown below for a repeating pattern, the block to be compared could be .



The block to be compared could also be chosen as  . This would again cover the pattern when repeatedly moved in steps along the string. If the beads are formed into a circle, they will show a rotationally symmetric pattern. If actual beads were used, instead of a drawing of beads, the comparison block of beads would of course not be laid on top of another block. It would be placed beside a corresponding block for the comparison.

There are many (imperfect) repeating patterns to be observed in nature. Two examples are bamboo stems and caterpillars. Often only a portion of the object shows the repeating pattern, as in the caterpillar where the middle segments are approximately of equal size and shape but the two ends are not. (See Figure 7.)

The activities in this unit will extend the concept of repeating patterns to poetry and music. Repeating sound patterns are common in both poetry and music.

## Bilateral Symmetry or Symmetry about a Line or Plane

Consider the letter A in Figure 8. Any line drawn perpendicular to the dotted line in the figure cuts the pattern in pairs of

Figure 8

points that are equidistant from the dotted line. For example, the points a and b in Figure 9 are at equal distances from the dotted line.

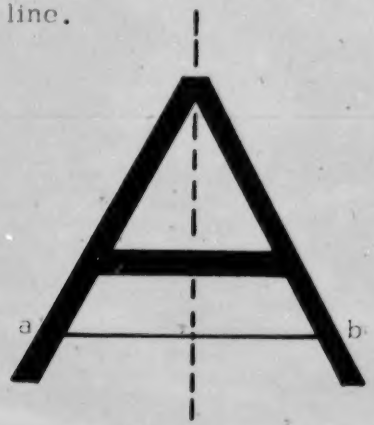



Figure 9

The letter A is said to exhibit symmetry about a line, or bilateral symmetry. The dotted line is called the line of symmetry or the axis of symmetry.

Any pattern through which a line can be drawn such that every line perpendicular to this line cuts the pattern into pairs of points equidistant from the line is said to exhibit symmetry about a line or bilateral symmetry.

A simpler but less exact statement of this idea is that for every point on one side of the line of symmetry there is a corresponding point on the other side. The paired points must be at equal distances from the line of symmetry. Figure 10 shows some almost exactly bilaterally symmetric patterns that occur in nature. There are tests one can use to determine whether or not a pattern possesses bilateral symmetry. Two of these tests are described below.

If a mirror is placed in a vertical position along the dotted line in Figure 8, the pattern viewed (half of A and its image in the mirror) coincides precisely with the original letter A. The mirror test consists of holding a mirror on the axis of symmetry to show that the pattern can be divided into two parts identical in size, one part being the mirror image of the other. In any bilaterally symmetric pattern, one half of the pattern is the mirror image of the other half. This is the reason that bilateral symmetry is also referred to as mirror symmetry. It is interesting to test some non-symmetric patterns with a mirror.

Bilaterally symmetric patterns may also be folded along the axis of symmetry to test for the coincidence of the two halves of the pattern. This gives another test, the folding test, for bilateral symmetry.

There are some differences in examining the symmetry of patterns on plane surfaces and of three-dimensional objects. Corresponding to the line of symmetry of plane patterns, bilaterally symmetric three-dimensional objects have a plane of symmetry. Every line perpendicular to this plane cuts the figure in pairs of corresponding points that are at equal distances from the plane. Obviously, a solid object cannot be folded to test for bilateral symmetry. However, a mirror can be placed along the plane of symmetry to show the existence of bilateral symmetry. The human face is an interesting object that has approximate symmetry with respect to a plane. Other objects that might be tested are a cone, a beetle, and toy figures of people or animals.

A picture of a three-dimensional object is, of course, a two-dimensional pattern. Thus the symmetry of a picture of a three-dimensional object may be tested by either the mirror or folding test. Generally the kindergarten and first grade activities will involve only two-dimensional patterns or pictures.

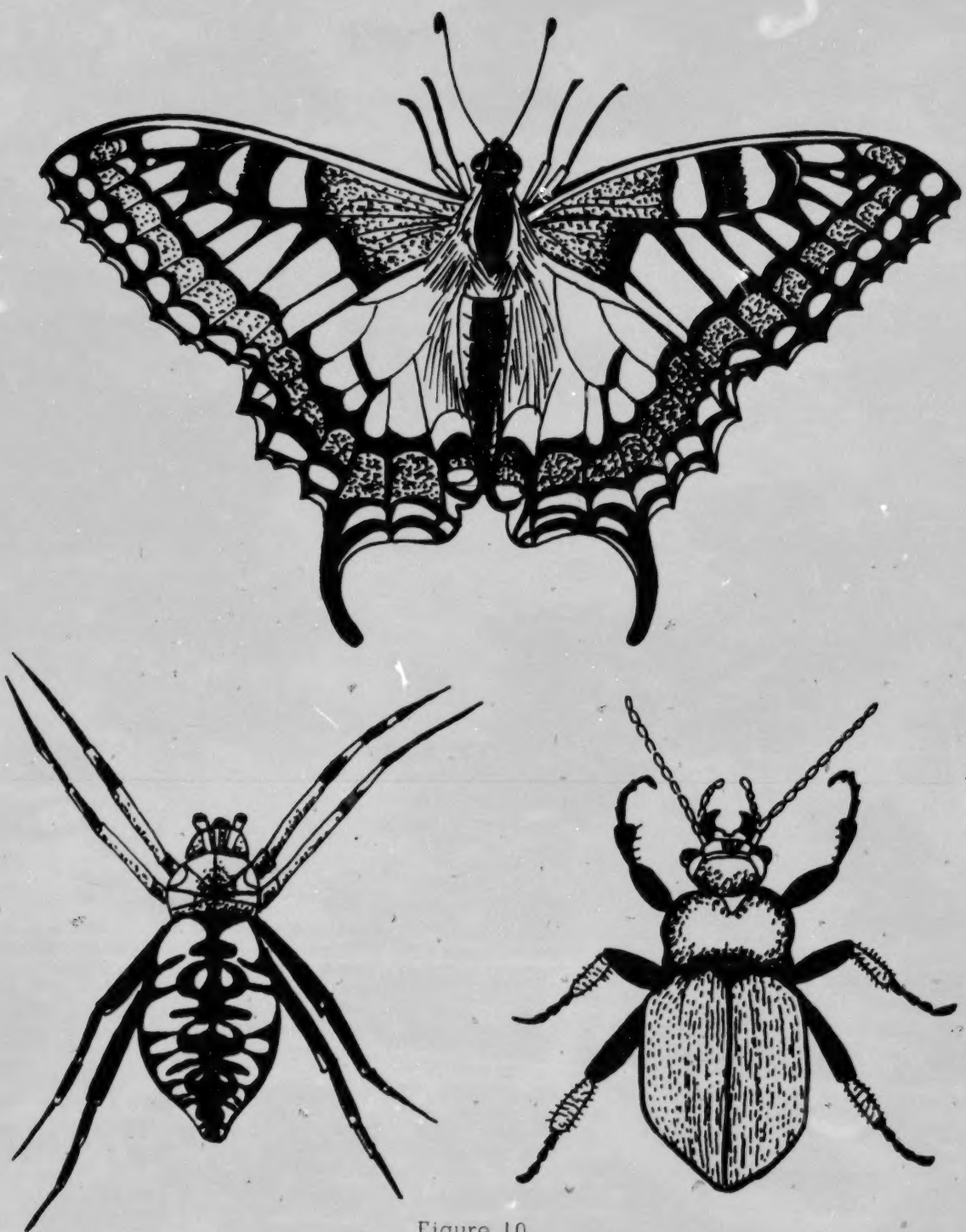


Figure 10
EXAMPLES OF BILATERAL SYMMETRY

## Other Symmetries

An object may display more than one of the symmetries that have been discussed. For example, a star such as the one in Figure 11 displays both rotational and bilateral symmetry. Notice that each of the five dotted lines is an axis of bilateral symmetry.

Figure 11

It is not necessary for a child to recognize more than one symmetry in an object, but it is important for you to be aware of the possibility of more than one, so that you may guide the children properly.

There are other symmetries which may be precisely described mathematically. You may be interested in noting these, but they will not be explicitly presented to the children. One such symmetry is a translation plus dilation. It is illustrated in Figure 12 by the shells of the common wentletrap, an auger, and the giant conch. Another symmetry which is a rotation plus dilation is illustrated in Figure 13 by the shells of the chambered nautilus and the sunburst carrier.

A more complicated kind of symmetry is found in certain two-dimensional patterns, which are called ornamental patterns. A common pattern of this kind is formed by fitting hexagons together as in a honeycomb or in a tiled floor. Ornamental patterns are also often seen in brick walls. (See Figure 14.) There is a precise mathematical theory underlying the ornamental pattern, but it is not necessary to understand this theory to appreciate or create ornamental patterns. Extremely intricate and beautiful tiled patterns have been produced by artists who instinctively used the laws of symmetry. The three-dimensional arrangements of atoms in a crystal are related to ornamental patterns.

There are other patterns possessing symmetries not discussed here. It is hoped that the child will recognize some of them in his environment. The child should not be expected to describe precisely what he sees, but simply to be aware of and enjoy the symmetric patterns about him.

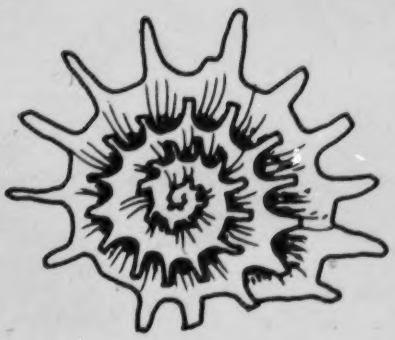
Common Wentletrap

Auger

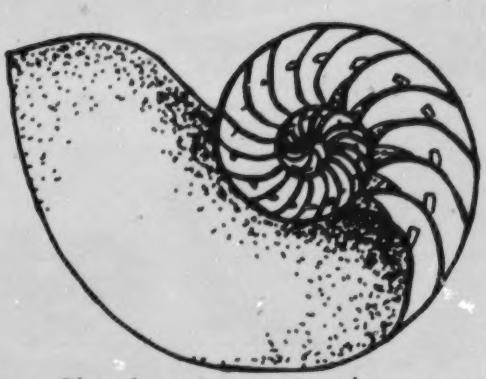
Giant Conch (Cross section)

TRANSLATION PLUS DILATION

Figure 12



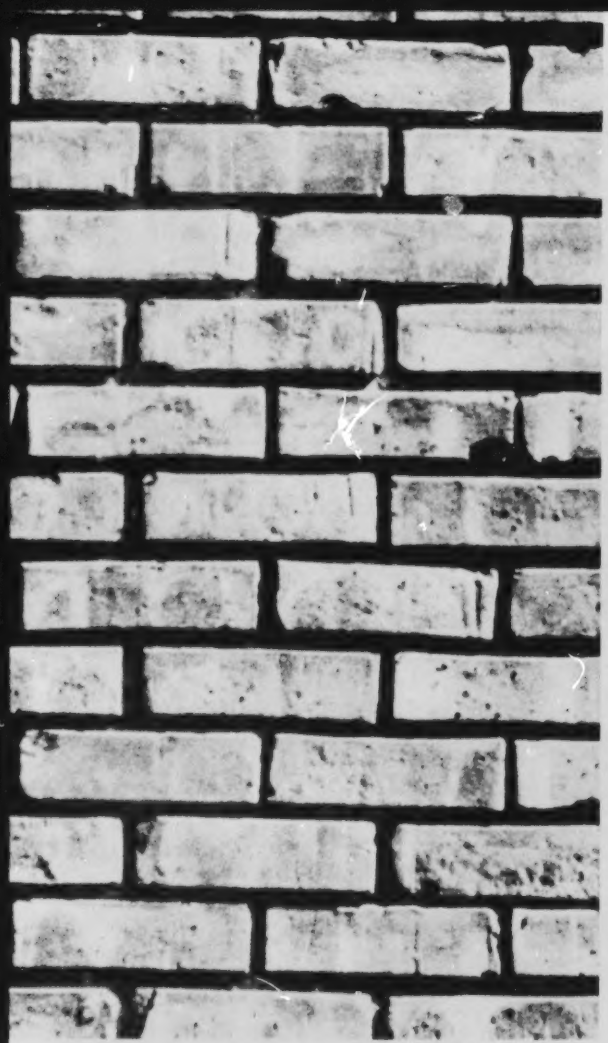
Sunburst Carrier



Chambered Nautilus (Cross section)

ROTATION PLUS DILATION

Figure 13



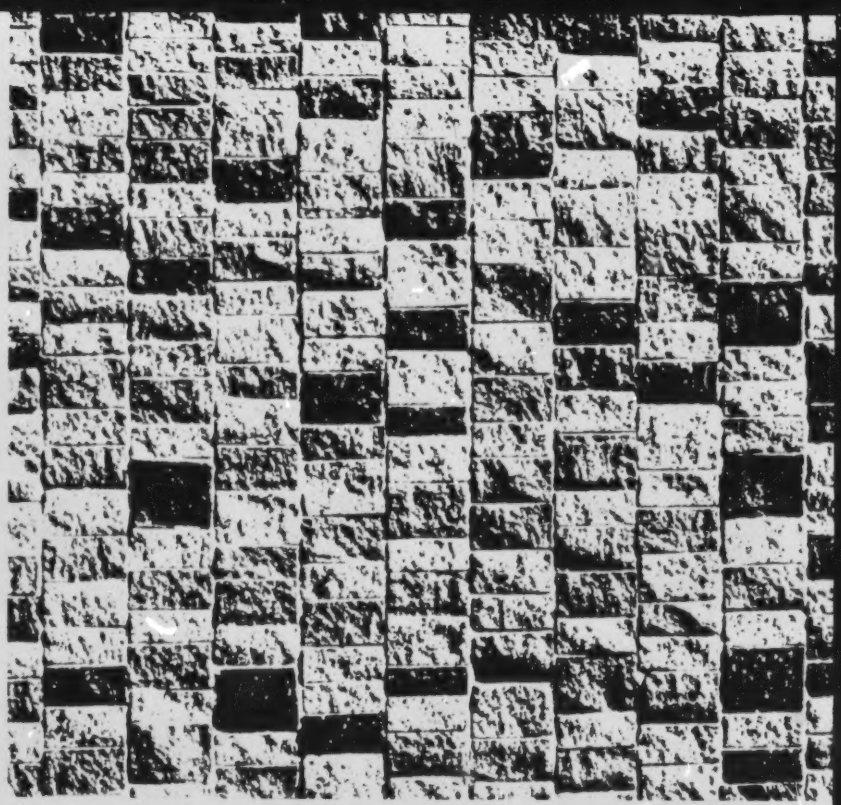
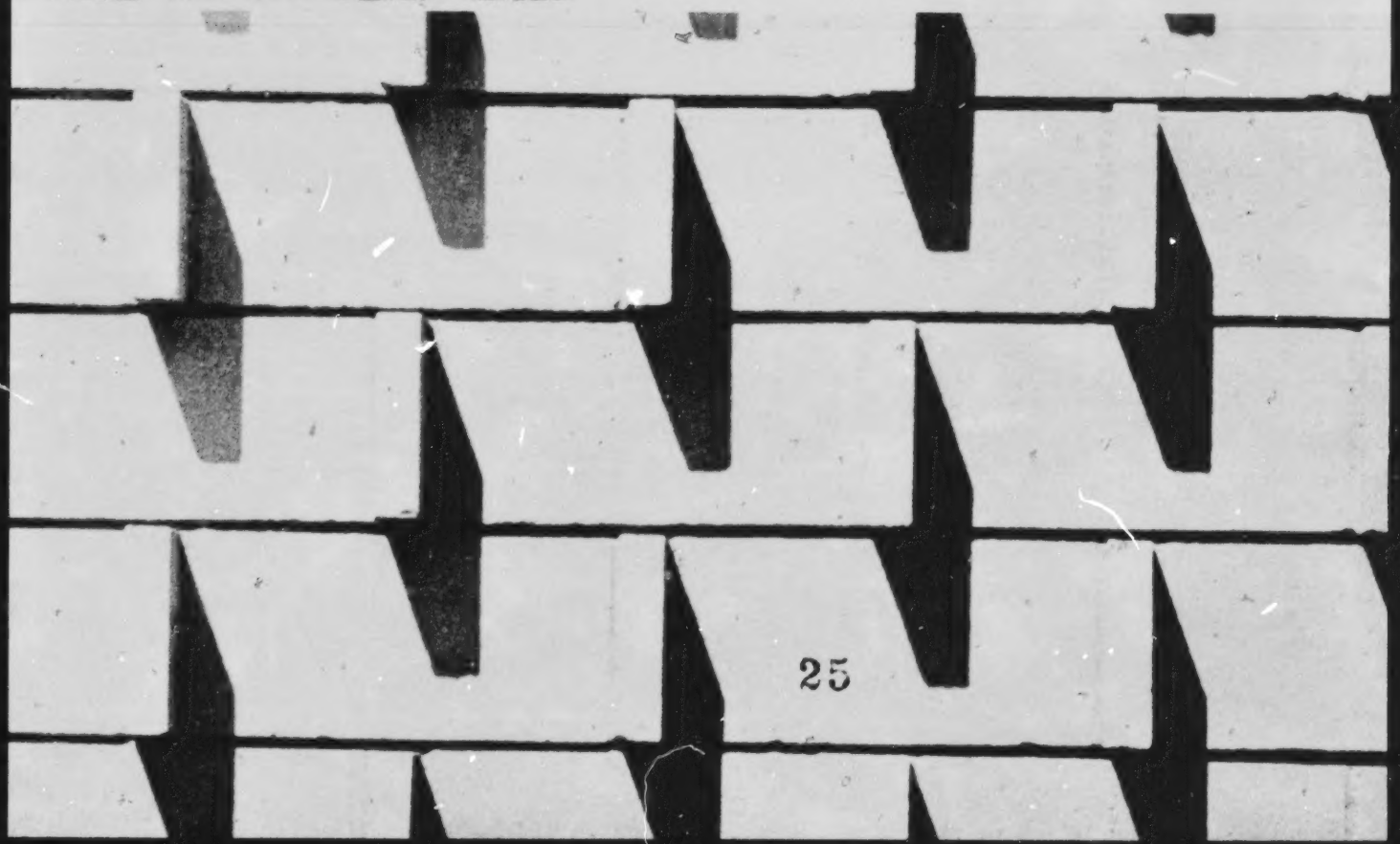


Fig. 14. SYMMETRY IN BRICK AND STONE
These three photographs show some of the many varied patterns used by architects and builders. Above left -- a wall built with plain bricks; above right -- the stone face of an office building; below -- shadows contribute their symmetry, too.



Summary

In summary the tests for rotational, translational, and bilateral symmetry are given:

1. Rotational symmetry test: an object is rotated about an axis and compared with a copy of the original object.

2. Repeating pattern test: a portion of a pattern is duplicated, moved along the pattern, and compared to parts of the pattern at equally spaced locations to determine coincidence.

3. Bilateral symmetry test:

   3-D: an object is compared to the half-object, half-image pattern obtained by using a mirror and is judged for coincidence.

   2-D: either the above mirror test is used, or the object is folded to test for coincidence of the two halves.

The use of a copy or replica of the object, or part of the object, in the tests eliminates the need for the tester to imagine the object or part to be moved and compared. You will probably carry out the tests mentally, but five- or six-year-old children may not be successful in making such abstract comparisons. If some children begin to form conclusions quickly concerning the symmetry of a particular object, this should not be discouraged. If they are correct, accept them with joy. If they are incorrect, a direct test for symmetry should be made, involving the concrete operations and comparisons necessary.

DIFFERENCES BETWEEN THE KINDERGARTEN AND FIRST-GRADE SYMMETRY UNI

If you look at copies of the kindergarten and first-grade symmetry units you will notice that the first four sections of each consider rotational symmetry, repeating patterns, bilateral symmetry, and symmetry in movement, poetry, and music. How then do they differ? Some of the ways are given in the following paragraphs.

An obvious difference is that the first-grade unit contains worksheets and an additional section. On many of the worksheets the children are asked to do more precise and accurate work than that asked of kindergarteners. The additional section for first grade considers crystals and ornamental patterns.

The first-grade unit considers the three basic symmetries in more depth. More complicated patterns are considered. At the end of Section 3 the first grader is asked to identify symmetries when more than one type are present in the same object.

Although the major emphasis is on observation and description in both units, the first-grade unit includes some work involving measurement and generalization. The first-grade unit is more quantitative. The children are asked questions such as "How many positions are there where the pattern looks just the same?" and "How far is the star from the line of symmetry?"

## COMMENTS ON THE TEACHING OF THIS UNIT

The lessons in this unit are usually planned to take one class period. The questions in capital letters that appear in some lessons are to be considered a guide to the type of questions you will use. Feel free to modify them and any other part of the unit to suit the needs of your own class.

The children should use and understand the concepts of the symmetries considered, but they should not be expected to become proficient in verbalizing them. They will not always be precise in their work. We hope that both you and your class will relax and enjoy this unit.

### Art Activities

The art activities suggested in this unit are an essential part of the children's investigations of symmetry. Through the free creation of patterns the children learn to recognize and appreciate the presence and absence of symmetry.

Each group of art activities can be used any time after the appropriate symmetry has been introduced. Many of the art activities can be modified and used for other types of symmetry. Direction should be kept minimal to encourage the originality a child may demonstrate when he is given a chance to use his own creative powers.

## SECTION 1 ROTATIONAL SYMMETRY

This first section of Unit 7 presents the concept of rotational or turning symmetry. Because the conceptual world of kindergarten children is based on material objects, the rotational symmetry of familiar blocks and other shapes is first investigated. Then the concept of rotational symmetry is abstracted and applied to other patterns. This reinforces the experiences with abstractions in the earlier MINNEMAST units.

Both man-made objects such as gears and natural objects such as starfish are investigated. Much freedom is given to the child when he creates rotationally symmetric designs in various media.

Black-Eyed Susan

## Lesson 1: TURNING BLOCKS

The concept of rotational symmetry is introduced in this lesson by activities that provide opportunities for muscular and perceptual learning.

MATERIALS

- 1 pair of triangular blocks for each child. The members of a pair should be matched for size, but different pairs can be of different sizes.
- masking tape for marking corners of the blocks
- additional pairs of blocks of various shapes for Activity B

PROCEDURE

Activity A

Give each child a pair of triangular blocks matched for size. One corner of the upper face of each block should be marked with masking tape. The tape helps in lining up the blocks and also marks one side as the front of the block. (Remember, in testing for rotational symmetry, the shape must not be turned over to bring the back side up.)

Have each child place one of his blocks on top of the other so that the taped corners match. Ask the class to hold the bottom blocks still and turn the top block until the two blocks again fit.

DID YOU TURN THE BLOCK ALL THE WAY AROUND? (No, only part of a full turn.)

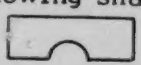
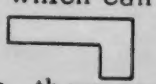
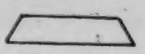
TURN THE TOP BLOCK MORE UNTIL THE BLOCKS FIT ONCE AGAIN.

DO THE TAPED CORNERS MATCH NOW? (No.)

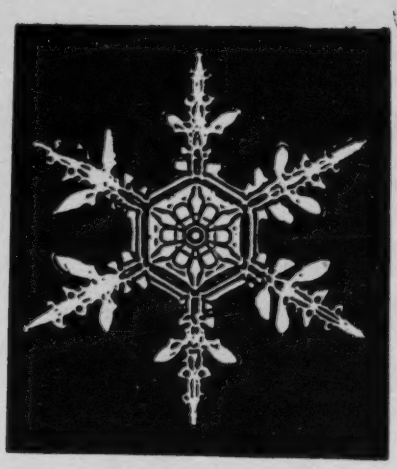
AGAIN TURN THE TOP BLOCK UNTIL THE BLOCKS FIT. DO THE TAPED CORNERS MATCH NOW? (Yes, the block has been turned all the way around to the place it was at the start.)

Repeat the above procedure if you wish. Explain to the children that when we can turn an object part way around and have it look the way it did before it was turned, we say the object has turning symmetry. If you wish, use the term rotational symmetry; many children enjoy using exact terminology.

Activity B

Provide matched pairs of blocks of various shapes, some rotationally symmetric. Square, rectangular, and circular blocks can be used for the symmetric pairs. You may have in the classroom blocks of the following shapes, which can be used for the nonsymmetric pairs:  ,  , , and  . Of course, there are many other suitable shapes.

The children can work individually or in groups with these blocks. Have them tell if their blocks have turning symmetry and show how they tested for it.



Snowflakes

## Lesson 2: STARS AND S-SHAPES

In this lesson more illustrations of rotational symmetry are given.

MATERIALS

- 1 blue and 1 yellow star transparency (For more exact cutting, stick both together with masking tape and cut at the same time.)
- overhead projector (very desirable but not necessary)
- 1 pair of stars and 1 pair of S-shapes cut from construction paper for each child

PROCEDURE

Activity A

It is assumed that you will have an overhead projector to use. If you do not, the transparencies may be viewed against a window pane by groups of children.

Project the yellow star and the blue star separately so that the children may observe the materials being used. Next superimpose and project the two star shapes together. Clearly demonstrate that when the blue and yellow stars match exactly, a green star is seen.

Now rotate the upper star slowly over the lower star. As this is done some yellow, some blue, and some green will be seen. After one-fifth of a full turn, the stars will again coincide, and an all-green star will be seen. The children should understand that this second coinciding of the patterns is a test result that tells that the star shape possesses turning (or rotational) symmetry. Explain that the bottom star serves as a record of the original position of the top star.

Activity B

Give each child a pair of stars and a pair of S-shapes. Cut these from the construction paper for the children, because

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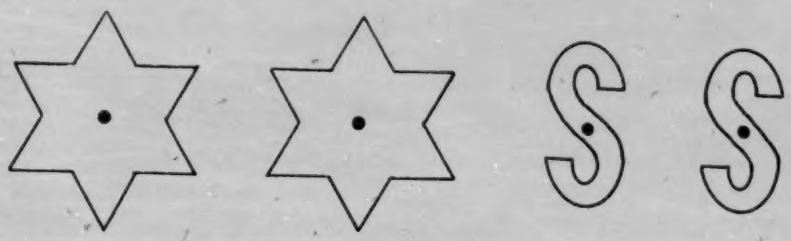
the inaccuracy of their cutting is apt to destroy the symmetry of the shapes. Show the children that there is a dot on one side of each shape. Explain that the dot marks the front or top side of the shape.



Allow the children to play with the shapes. They will probably fit them together and rotate them. They may notice that their paper stars have six points whereas the transparent stars have only five. Questions similar to the following would be appropriate to ask individual children.

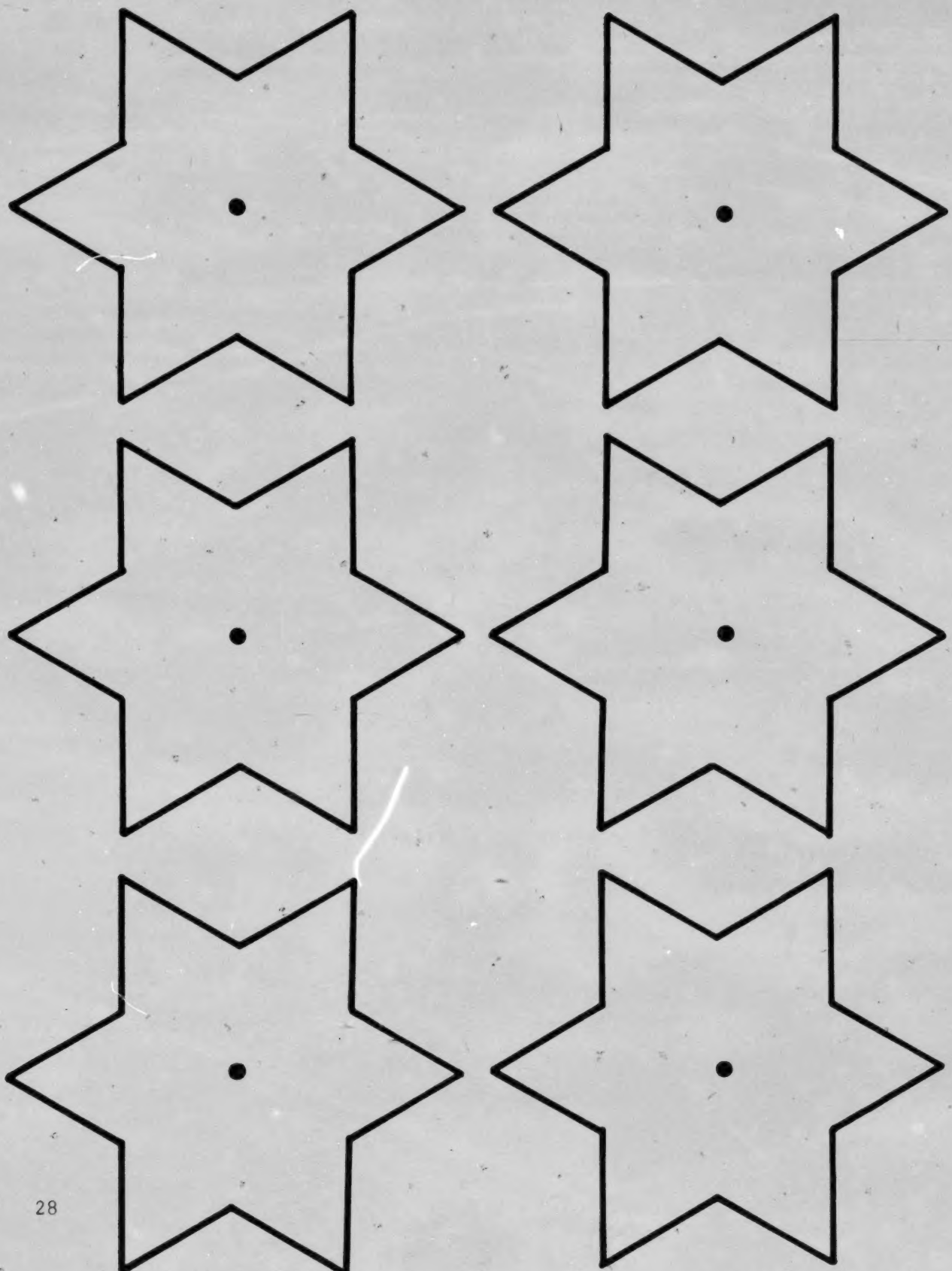
DOES THE STAR HAVE TURNING SYMMETRY? (Yes.)

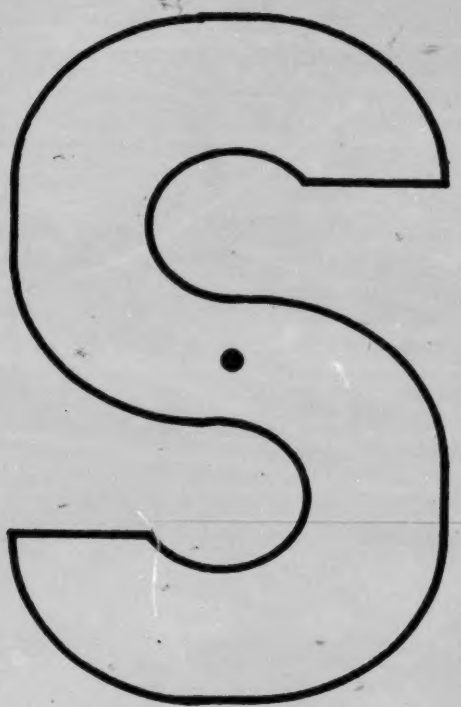
DOES THE S-SHAPE HAVE TURNING SYMMETRY? (Yes.)

HOW CAN YOU SHOW THIS? (We can turn one shape part of a turn and see that it again fits the first shape.)

IF YOU TURN THE S-SHAPE OVER -- WITH THE PRINTED DOT DOWN -- WILL THE S-SHAPE FIT? (No.)

Save the paper shapes for use in Lesson 8.





These patterns are provided on construction paper with the materials that accompany this manual.

## Lesson 3: WHICH OBJECTS HAVE TURNING SYMMETRY?

In this lesson the children examine various objects for rotational or turning symmetry. Some of the objects will be three-dimensional. Some will be only approximately symmetric objects found in nature.

MATERIALS

- an assortment of rotationally symmetric objects. Some examples are certain flowers (real or artificial), certain shells, an apple or orange sliced across, starfish, gears, wheels, nested blocks, and cardboard shapes (H, 8, +).

- an assortment of objects that are not rotationally symmetric. Some examples are most leaves, plastic figures of trees and animals, certain blocks, certain shells, scissors, and cardboard shapes (♡, ☾, 7).

PROCEDURE

Prepare a shape table in the classroom. Place on the table some rotationally symmetric objects and some that are not rotationally symmetric. Most of the objects should be presented in pairs to help the children compare the pattern of the object before and after it is turned. Some single objects should also be used. Place other rotationally symmetric and non-symmetric objects about the room. Suggestions for the objects are given in the materials list.

Have the children look on the shape table for objects possessing rotational symmetry. They should test an object by seeing whether it looks just the same after it has been given a partial turn. The second object of each pair is used as a reference unit in this comparison, or in the case of single objects the original pattern can be remembered. Some children may comment on the fact that the turning symmetry of natural objects is usually not exact.

After the objects on the shape table have been examined, ask the children to think of other objects out-of-doors or at home that have turning symmetry. Also ask them to discover whether there are any more objects with turning symmetry in their classroom.

## ART ACTIVITIES FOR SECTION I: ROTATIONAL SYMMETRY

The child's concept of rotational symmetry can be reinforced and expanded by his experiences with some of the art activities suggested here. Select any you wish, vary them as you wish, and use them at any time after Lesson 2 has been taught.

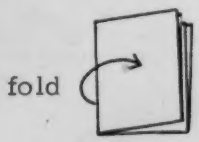
Two toys that mechanize the drawing of geometric patterns, most of which are rotationally symmetric, are listed below. A large variety of patterns may be produced fairly easily. (A ball point pen usually gives better results with these toys than does a pencil.)

Magic Designer 8245 (Lakeside Toys, Division of Lakeside Industries, Minneapolis, $4.95).

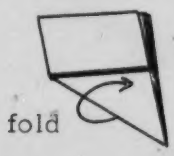
Spirograph No. 401 (Kenner Prod. Co., Cincinnati, $3.98).

Activity A: Design Silhouette

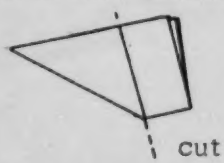
1. Fold a sheet of typing paper into quarters.



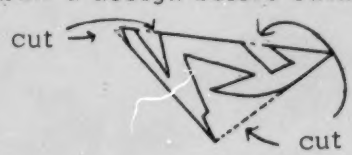

2. Fold again along the diagonal.



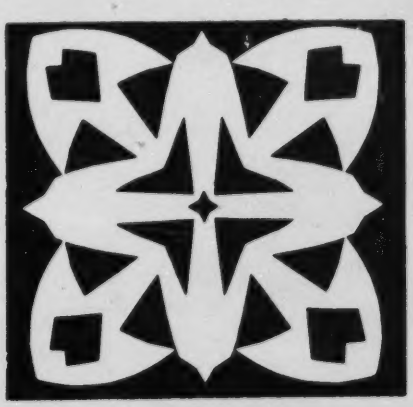

3. Cut as shown to form a square.

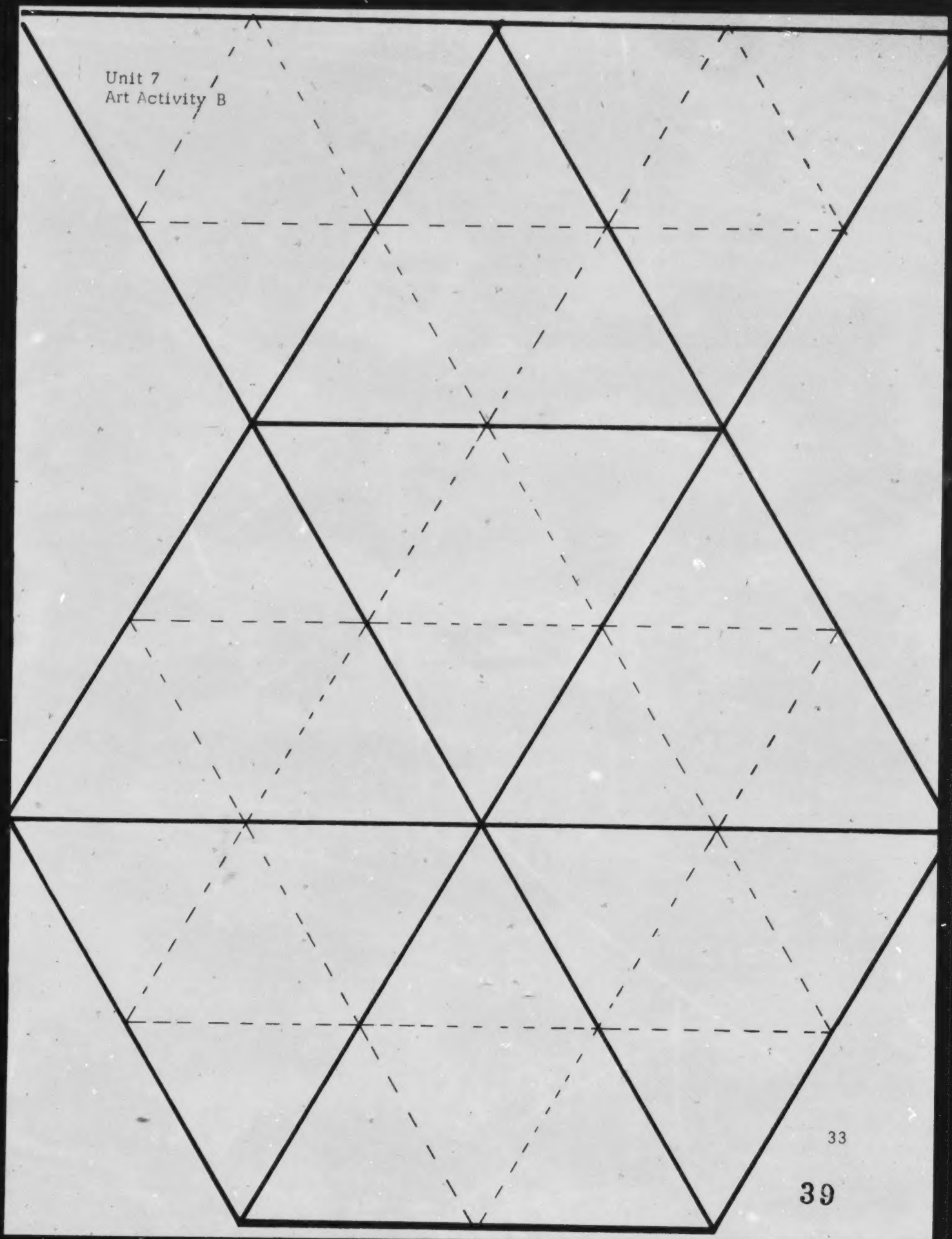



4. Cut out design from folded paper. (If you wish, you may draw a design before cutting.)




5. Unfold design and paste on black construction paper.




Unit 7
Art Activity B

Activity B: Triangle Flower

1. Distribute the sheet with equilateral triangles printed on it.

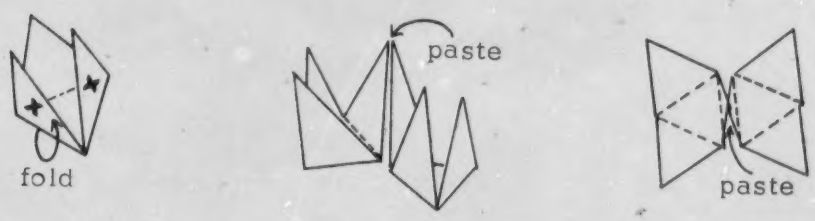
2. Cut along the heavy lines.

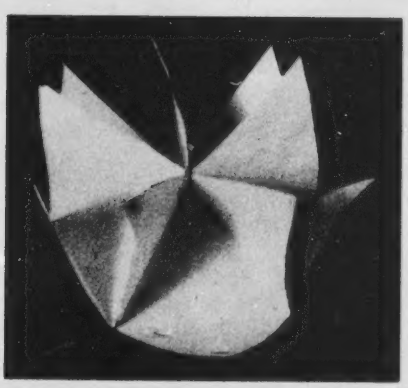
3. Fold up along the dotted lines. You now have four smaller triangles marked off by folds in each large triangle.

4. Put paste on the two outer triangles marked with X's in the first diagram.

5. Take two large triangles and press together one pasted surface of each. Thus each triangle will have one corner free and will be attached to two other triangles by its other two corners.

6. The last triangles should be attached to the first, to form the flower.

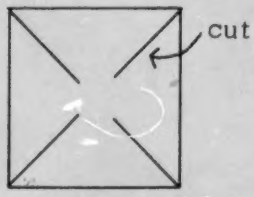


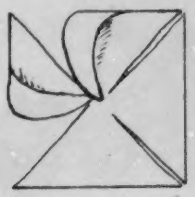
Activity C: Pinwheels

Materials needed are construction paper, tacks, and pencils with erasers.

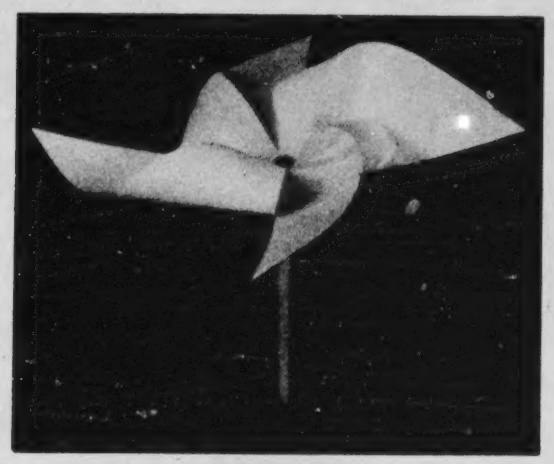
1. Take a square sheet of construction paper and cut from the corners toward but not all the way to the center. You now have eight points.




2. Bring every other point, one by one, to the center and fix them there with a tack.

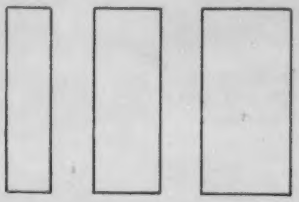


3. Push tack into the eraser at the top of a pencil.

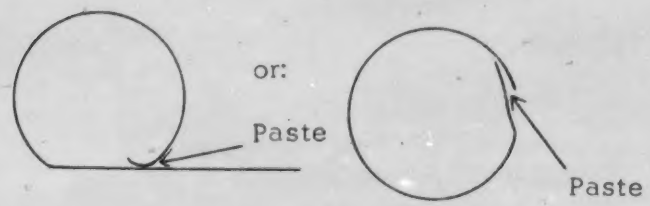


Activity D: Loop-de-doos

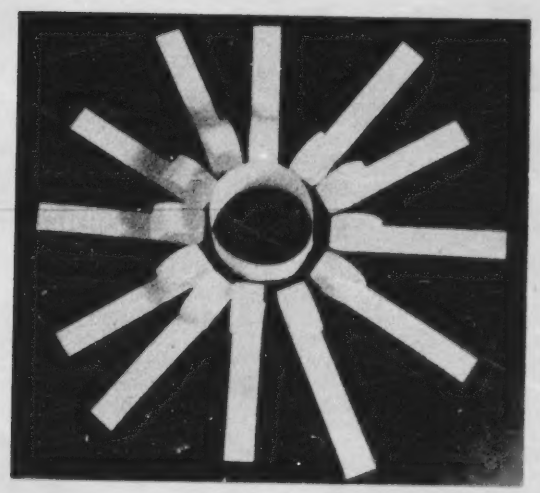
1. Cut strips of colored construction paper into 1/2", 3/4" and 1" widths.



2. Make loops of assorted sizes, some with tails.




3. Arrange design on construction paper and paste.



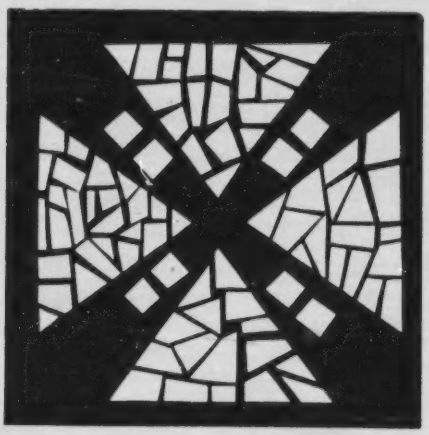
Activity E: Paper or Yarn Mosaics

You will need tagboard, scraps of colored paper or yarn, and white all-purpose adhesive such as Elmer's or Fuller's.

1. Draw rotationally symmetric designs on tagboard.

2. Fill in areas of the design with quarter-inch (or larger) scraps of colored paper and paste them on the tagboard.

3. For yarn mosaics, fill the design with short pieces of colored yarn, fixing them in place with some all-purpose glue.



Variation

Feathers, leaves, flowers, shells and various kinds of macaroni may also be used for mosaics.

## SECTION 2 REPEATING PATTERNS

In this section the children study, create, and enjoy finite repeating patterns. If the finite patterns are thought of as being infinite, they will exhibit translational symmetry. Although the concept of translational symmetry is discussed in the introduction to the unit, it is not presented to the children in kindergarten.

The children work with real objects and concrete operations throughout the section. They will learn that any pattern which can be made by repeating a part of the pattern at equal intervals along a straight line is a repeating pattern. Repeating patterns are often more easily recognized in man-made designs than in the world of nature. However, the approximate symmetry of natural objects such as caterpillars is also observed.

Baird's Swallowtail (larva)

## Lesson 4: PAPER-DOLL CHAIN

This lesson introduces the idea of a pattern in which one part is repeated a number of times along a line. The lesson is a short demonstration. You may choose to combine it with Lesson 5 where the children make their own repeating patterns.

MATERIALS

- paper-doll chain transparency (blue)
- paper-doll transparency (yellow)
- overhead projector (desirable, but not required)

PROCEDURE

As in Lesson 1, it is assumed that you will be able to project the transparencies. However, the transparencies may be viewed against a window.

Project separately the paper-doll chain and the single paper doll. Next project the transparencies together with the single figure on top. Bring the single figure into coincidence with a figure in the chain. This figure will now appear green. Move the single figure to an adjacent figure in the chain. This figure will now appear green. Continue moving the single figure until each figure in the chain has been covered.

A class discussion should bring out the following ideas: A green figure shows that the yellow paper doll fits exactly over each blue paper doll. The fact that the yellow doll fits exactly over each blue doll shows that each blue doll has the same shape. The blue doll shapes make a repeating pattern.

Any art activities suggested for this section may now be used.

## Lesson 5: KEEP THE PATTERN

The children are here given an opportunity to create designs with various shapes and colors. The only restriction is that their basic designs are to be repeated. By making their own repeated patterns, the children demonstrate their grasp of the concept and have it reinforced.

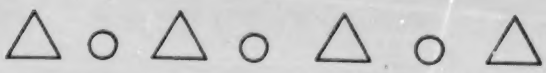
MATERIALS

- sets of objects from which patterns can be made. Some examples are: property blocks, beads to string, flannel board shapes, and pegs for pegboards.

PROCEDURE

Activity A

Place a sequence of shapes on the chalk or flannel board, or make a similar pattern with other materials, e.g:



CAN ANYONE ADD ANOTHER SHAPE AND KEEP THE PATTERN REPEATING? (Yes.)

Have a volunteer do this several times.

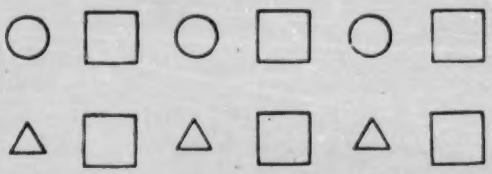
Now have the children work singly or in small groups. Provide sets of objects with which they can arrange repeating patterns. Tell them to make any patterns they wish by repeating their basic designs over and over. Examples:

| Basic design | Basic design repeated |
|---|---|
| 0 0 / | 0 0 / 0 0 / 0 0 / 0 0 / |
| + 0 + | + 0 + + 0 + + 0 + + 0 + |

An element of competition may be inserted by having the members of a group take turns in adding to the design. The "keep the design" rule should be understood by all.

Activity B (Optional)

After a child or group of children has made a repeating pattern, suggest that the pattern be changed by following a definite rule. Suppose the pattern has been made with property blocks. The first rule might be that all small circular blocks be changed to small triangular blocks of the same color. (○ ⟶ △) For example:



DO WE STILL HAVE A REPEATING PATTERN? (Yes.)

Ask the children to suggest other rules to use. They may be simple, such as (thick ⟷ thin) or (red ⟶ blue), or quite complex, such as (thick ⟷ thin and red ⟶ yellow ⟶ green ⟶ blue ⟶ red).

## Lesson 6: FIND THE REPEATING PATTERNS

The children look for repeating patterns in the classroom. Many of the patterns are "planted" for them to find.

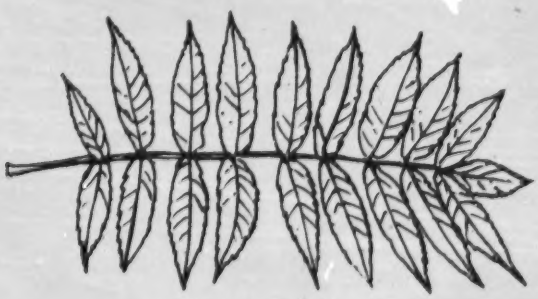
MATERIALS

- an assortment of objects, pictures, and designs that exhibit repeating patterns. Some examples are a caterpillar, a compound leaf, animal tracks, a fence, a chain (metal or paper), a strip of lace, a patterned belt, a necklace, a wallpaper border, smocking on a dress, and a piano keyboard.
- an assortment of objects, pictures, and designs that have several parts that do not repeat exactly. Some examples are a row of plastic figures that are only approximately alike, a paper chain with randomly colored links, a strip design that you have made for this lesson.

PROCEDURE

Place on the shape table, bulletin board, and in the classroom some objects, pictures, and designs that exhibit repeating patterns and some that do not. Suggestions for these are given in the materials list. Any completed art projects for this section can be included.

Ask the children to find examples of repeating patterns in the room. In each case the child should see that the pattern can be made by repeating one part of the design at equal steps along a straight line. When considering some objects, e.g., a caterpillar, the two ends must be excluded. Ask the class to tell about objects at home or out-of-doors that have repeating patterns.



## ART ACTIVITIES FOR SECTION 2: REPEATED PATTERNS

Various art activities that develop the idea of repeating patterns are described here. They may be used as you choose after Lesson 4 has been presented.

Activity A: Paper Chains

1. Select two colors of construction paper (e.g., red and black).

2. Cut strips 1" x 5", making three times more strips of one color than the other.

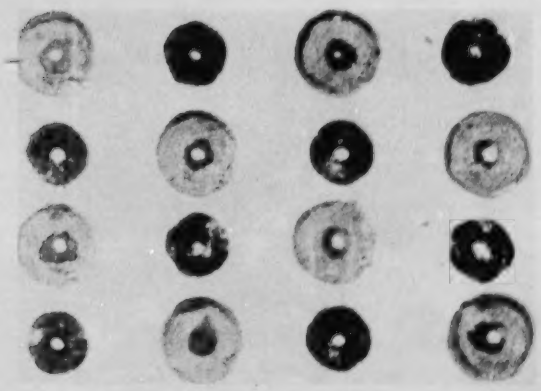
3. Make loop chains, following a repeated pattern (3 red loops, 1 black, 3 red, 1 black, etc.)

4. Variations in the paper chains can be made by cutting paper strips 1/2" by 6" and twisting each strip before pasting.

Activity B: Printing

You will need construction paper, tempera paint and brushes. Have the children look for small objects having a side flat enough to be suitable for stamping patterns. Encourage them to look for unusual objects that will make interesting designs. Examples are: spool, nail, plastic or tin cover, bottle cap, rim of a can, piece of wood, block, eraser, fork tines, thumb or finger.

Cover a surface of the selected object with paint and press it against the paper. A series of impressions side by side will make a repeated pattern. Another way of printing is to wrap string around a block of wood, cover string with tempera and press on paper for print.



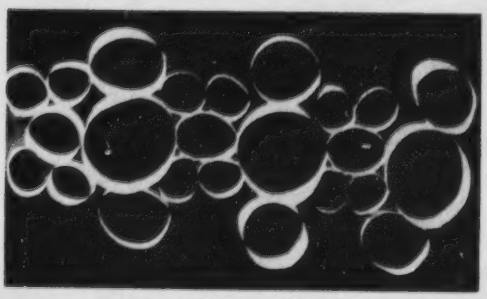
Activity C: Glued-Object Patterns

Repeating patterns can be made by using a number of small similar objects and pasting or gluing them in a row on tagboard. Examples are: small pebbles, Popsicle or craft sticks, leaves, pieces of construction paper, or stickers. (If stickers are used, the stars should be larger than those usually on hand in the classroom.)

Activity D: Papercraft

Many of the activities in Section I can be used here to illustrate repeated patterns. For example, loop-de-doos can be pasted down in rows.

Small rotationally symmetric designs can be pasted together in rows.



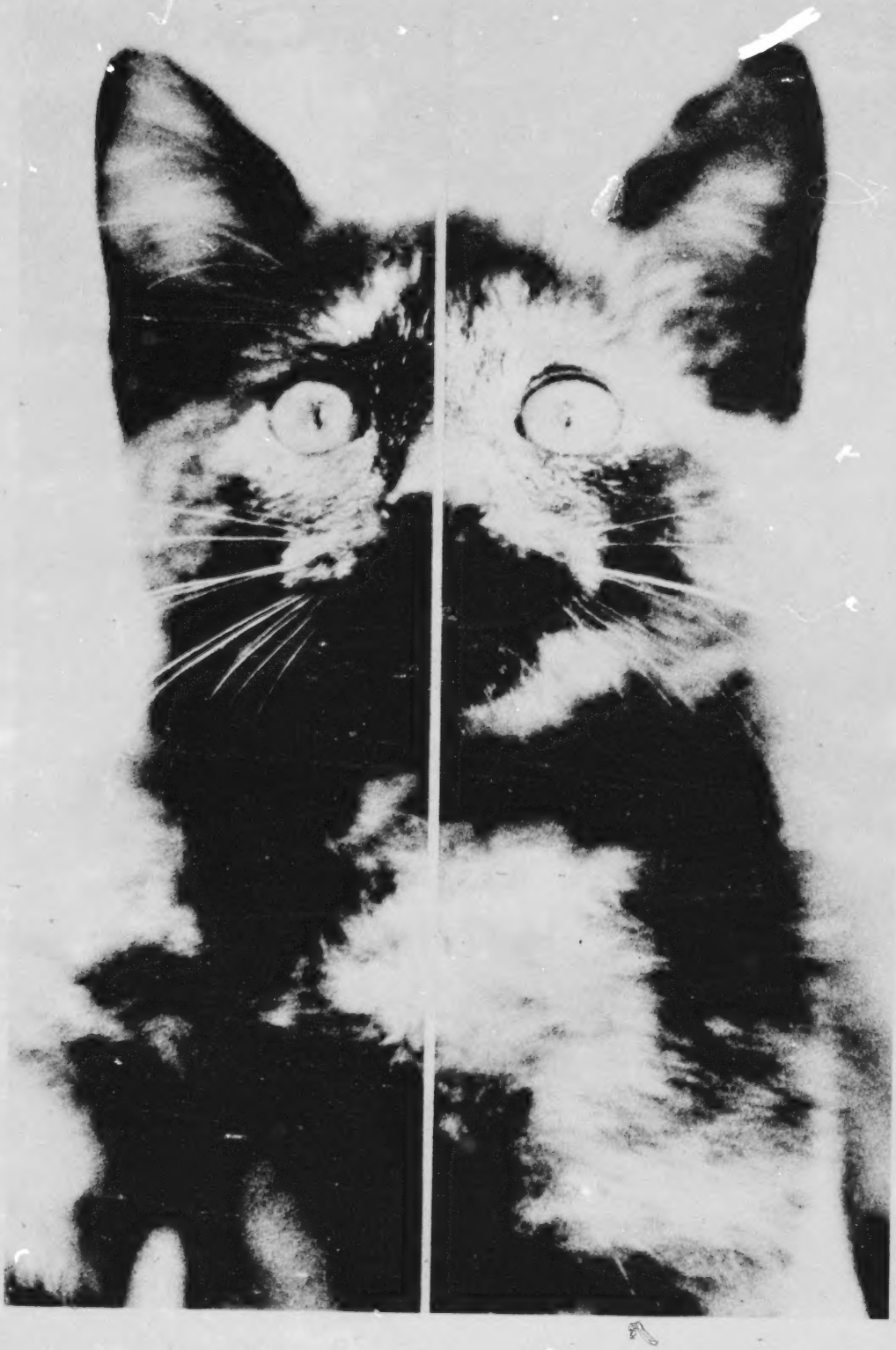
## SECTION 3 BILATERAL SYMMETRY

This section of Unit 7 introduces the concept of bilateral symmetry. Once again the children work with two-dimensional patterns, man-made and natural objects, and art projects. They are expected to begin to understand that a pattern that is unchanged by a reflection about an axis is bilaterally symmetric. However, it is not expected that they will be able to express the concept in these words.

Bilateral symmetry is sometimes called folding symmetry or mirror symmetry because two of the tests for this symmetry use folding and mirror images. Some of the patterns in this section will have both bilateral and rotational symmetry. This may be noted by some alert children, but the lessons consider only bilateral symmetry.

Moth

54



## Lesson 7: FACES AND HANDS

This lesson introduces the concept of bilateral symmetry.

MATERIALS

- 2 pictures of cat's face printed in this manual

PROCEDURE

Activity A

Show the picture of the complete cat's face to the class. Point to appropriate parts as you ask the following questions.

DOES EACH SIDE OF THE CAT'S FACE HAVE AN EYE? (Yes.)

DOES EACH SIDE OF THE CAT'S FACE HAVE AN EAR? (Yes.)

IS EACH EAR THE SAME DISTANCE FROM THE MIDDLE OF THE FACE? (Yes. Trace the center line of the face with a finger.)

DOES THE CAT'S FACE HAVE A KIND OF BALANCE? FOR EACH PART ON ONE SIDE IS THERE A PART LIKE IT ON THE OTHER SIDE? (Yes.)

ARE THE TWO HALVES OF THE FACE EXACTLY ALIKE? (There may be either yes or no answers from the children. Don't tell them yet whether their answers are correct.)

Show the class the second cat's face cut into two parts along the white line but held side by side. Suggest that if the two sides of the face were exactly alike you could interchange them and the face would look just the same as before. Demonstrate this.

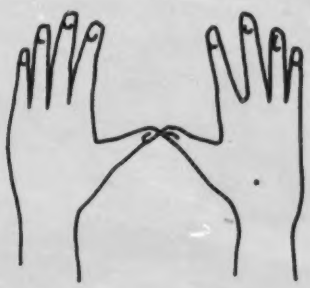
DOES THE CAT'S FACE LOOK THE SAME NOW? (No.)

ARE THE TWO HALVES OF THE FACE EXACTLY ALIKE? (No.)

Explain that the two halves of the cat's face have the same parts but that the positions of the parts are reversed. For example, on one side the ear is to the left of the eye and on the other side the ear is to the right of the eye. Place the two halves in their original positions and allow the children to use them later during free time.

Activity B

Have each child place his hands on the floor or table in front of him. The palms should be down with the thumbs just touching.



ARE YOUR HANDS EXACTLY ALIKE? (No, because the fingers of one hand are in opposite order to those of the other hand.)

Ask each child to put his palms together.

DO YOUR HANDS JUST FIT TOGETHER? (Yes.)

Explain that the way the fingers fit against each other shows that the parts of the two hands are alike but are arranged in opposite order. You may now tell the class that both the cat's face and the children's hands have a special kind of symmetry called bilateral symmetry.

Activity C (Optional)

To begin a class discussion you may ask the following question:

DO YOU THINK YOUR FACE HAS BILATERAL SYMMETRY? (Yes.)

## Lesson 8: THE FOLDING TEST FOR BILATERAL SYMMETRY

The experiences of Lesson 7 are extended to develop a folding test for the presence of bilateral symmetry in a pattern. The children are encouraged to manipulate, observe, and experiment with transparent acetate butterfly patterns and paper shapes.

MATERIALS

- transparent yellow butterfly
- transparent blue half-butterfly
- overhead projector (desirable, but not necessary)
- 1 paper star and 1 paper S-shape for each child (saved from Lesson 2)

PROCEDURE

Activity A

As in earlier lessons where acetate patterns are used, it is most effective to show these with an overhead projector. However, it is possible to view the patterns against a window if a projector is not available.

Project the yellow butterfly.

DO THE TWO SIDES OF THE BUTTERFLY LOOK ALIKE IN SOME WAYS? HOW? (Yes. "The two wings are the same size" is one of many possible responses.)

ARE THE TWO SIDES OF THE BUTTERFLY EXACTLY ALIKE? (No. Most children should be prepared by Lesson 7 to give this answer, but do not tell them whether they are correct until the end of this activity.)

Project the blue half-butterfly beside the yellow butterfly. Suggest that it can be used to see if the two sides of the yellow butterfly are exactly alike. Move the blue figure until it coincides with one side of the yellow butterfly. That side now appears green.

DOES THE BLUE HALF-BUTTERFLY FIT EXACTLY OVER HALF OF THE YELLOW BUTTERFLY? (Yes.)

Slide the blue half-butterfly to the other side of the whole butterfly.

DOES THE BLUE HALF-BUTTERFLY FIT EXACTLY ON THIS HALF OF THE YELLOW BUTTERFLY? (No.)

HOW CAN I MAKE IT FIT? (Turn it over.)

Turn the half-figure over, making the motion clear to the class.

DOES THE BLUE HALF-BUTTERFLY FIT EXACTLY OVER THIS HALF OF THE YELLOW BUTTERFLY NOW? (Yes.)

WOULD THE TWO HALVES OF THE YELLOW BUTTERFLY FIT EXACTLY IF ONE OF THEM WERE FOLDED OVER THE OTHER? (Yes.)

Explain that because the two halves of the butterfly pattern fit exactly together when they are folded, we know that the butterfly is <u>bilaterally symmetric</u>.

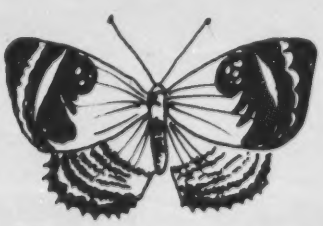
Activity B

Give each child a construction paper star and an S-shape saved from Lesson 2.

HOW CAN WE TELL IF THESE SHAPES HAVE BILATERAL SYMMETRY? (We can use the folding test.)

Let the children fold their shapes with a minimum of help. They will discover that the stars are bilaterally symmetric and the S-shapes are not.

Art activities provided at the end of this section may now be started.



## Lesson 9: MIRROR TEST

The mirror test for bilateral symmetry is taught in this lesson. Both two- and three-dimensional objects are examined, but the emphasis is on the testing of two-dimensional patterns. The lesson will probably take two days.

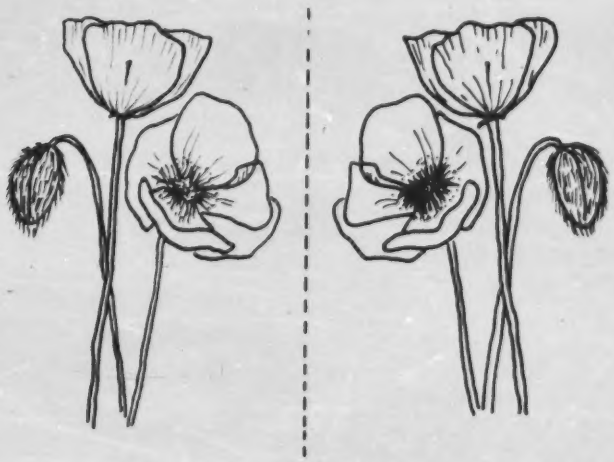
MATERIALS

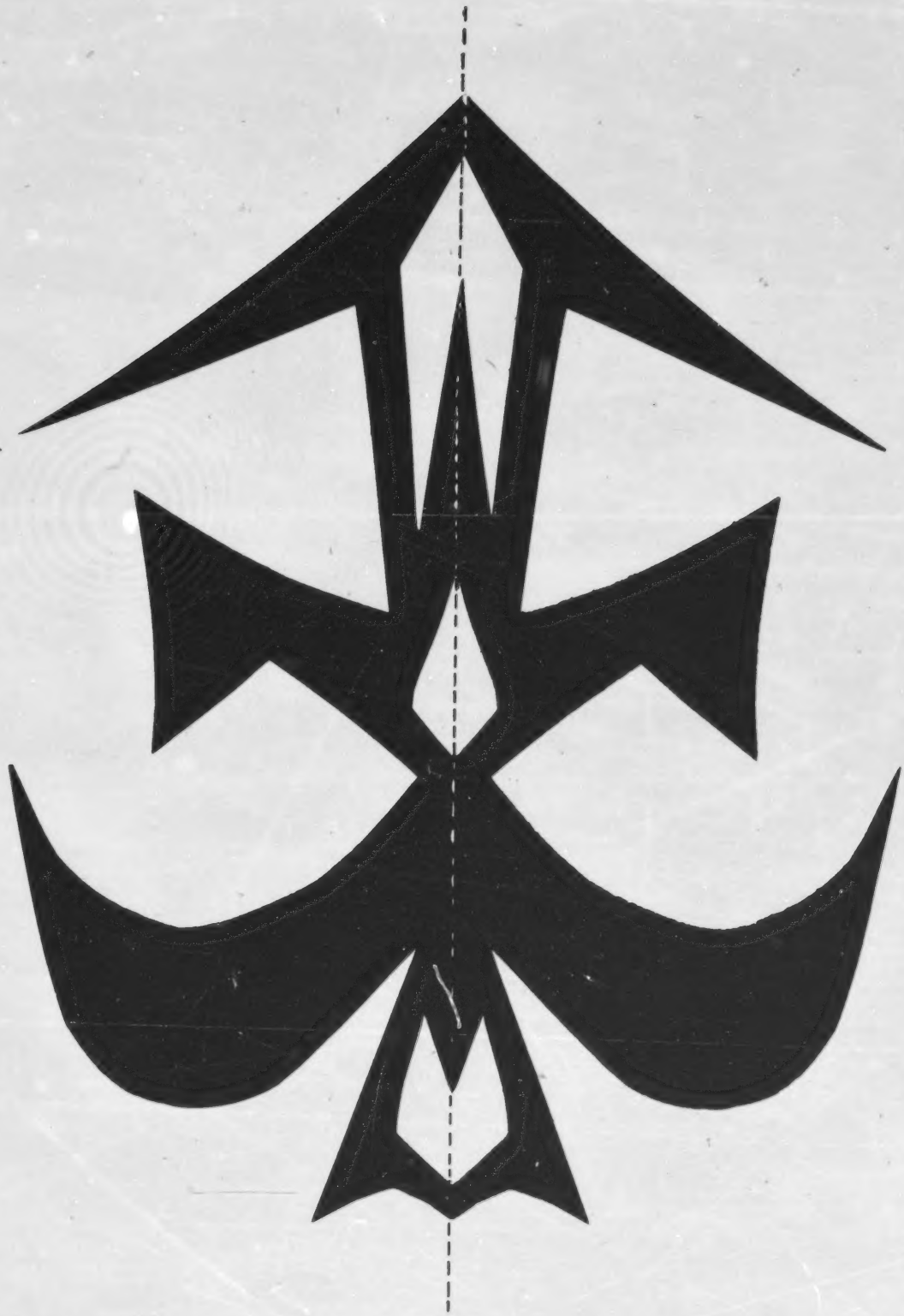
- Design Sheet 1 and 2 for each child
- small mirror for each child
- one larger mirror (If necessary, this may be made from two small mirrors.)
- assorted small solid objects to be tested with mirrors

PROCEDURE

Activity A

Place the figure on page 59 so that the class can see it. Hold the larger mirror just above the line of symmetry of the figure. Have a child tell what he sees as he looks into the mirror. Bring out the fact that he sees the pattern just as it was without the mirror. This is another test to show that a pattern has bilateral symmetry. This demonstration is intended to show the general procedure to the children. They will learn much more when they use their own mirrors.



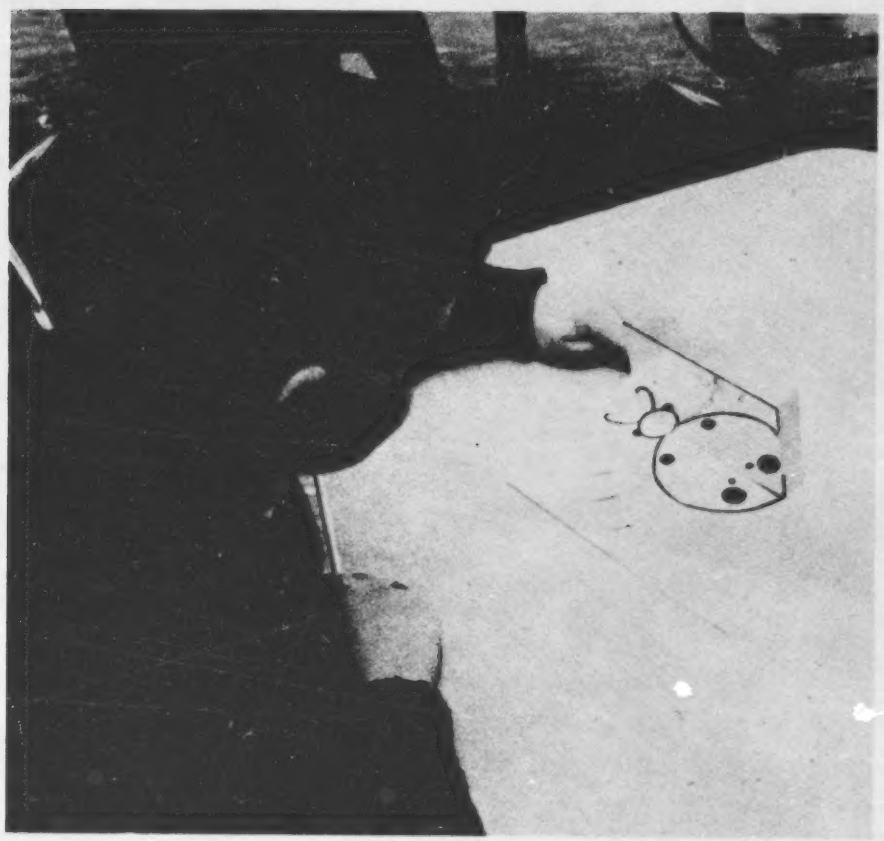


62

Activity B

Give each child a small mirror and a copy of Design Sheets 1 and 2. It is important that each child make his own unhurried investigations of the patterns on the design sheets and of any other patterns he wishes. However, a brief reminder of the mirror test is in order. If there exists a position for the mirror that will give a view of the pattern just as it was before, the pattern is bilaterally symmetric. Two of the four designs provided are bilaterally symmetric, two are not.

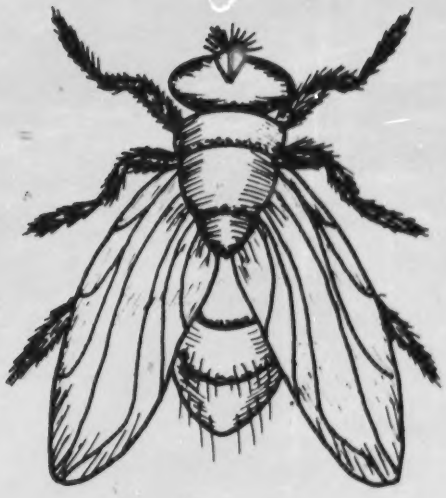
Some patterns are bilaterally symmetric with respect to more than one line. It is fine if children notice this, but it should not be required of them.



Design Sheet 1
Unit 7

64

Design Sheet 2
Unit 7



65

Activity C

Later, perhaps on the next day, show the class a two-dimensional symmetric pattern or picture. (The cat's face can be used.)

HOW CAN WE TEST THIS PATTERN TO SEE IF IT HAS BILATERAL SYMMETRY? HOW CAN WE TEST IT ANOTHER WAY? (With the mirror or folding test.)

Demonstrate the two tests.

Next show the class a bilaterally symmetric three-dimensional object -- a vase, an hour glass, or a doll are possibilities.

DO YOU THINK THIS IS BILATERALLY SYMMETRIC? CAN WE TEST IT WITH A MIRROR? (Yes.)

Make the test with the larger mirror.

CAN WE TEST IT BY FOLDING? (No.)

A discussion can bring out the fact that solid objects cannot be folded, but can be tested for symmetry with a mirror. It is not intended that you mention the fact that bilaterally symmetric three-dimensional objects have a plane of symmetry rather than just the line of symmetry of a two-dimensional pattern.

Provide a number of small solid objects for the children to view with the small mirrors. Possible objects are plastic toy figures, blocks, cookies, combs, parts of construction sets, and flowers. They may also look at their fingers, hands, and feet.

## Lesson 10: KEEP THE SYMMETRY

In this lesson the children arrange objects into bilaterally symmetric patterns. This should be an enjoyable way to reinforce the concepts of this section. In Activity B the changing of patterns by a specific rule provides an elementary background for the mathematical concept of mapping, which will be developed in later grades.

MATERIALS

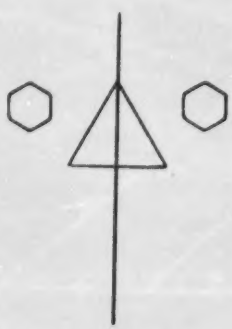
- flannel board with sets of objects
- sets of objects from which patterns can be made. Some examples are property blocks, checkers, paper clips, flannel shapes, pegs for pegboards.
- Design Sheet 3 -- 1 for each child
- large mirror, about 8" x 10" (optional)

PROCEDURE

Activities A and B of this lesson are similar to Lesson 5 and hence the children should need little explanation here. The designs discussed below may be made of any small objects. Several sets are suggested in the materials list. The children may enjoy testing their patterns with a large mirror.

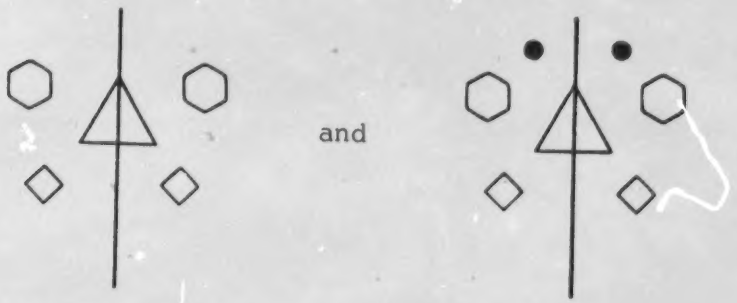
Activity A

Make a simple, bilaterally symmetric pattern on the flannel board. You may mark the line of symmetry with a string. A possible pattern is:



CAN ANYONE ADD TO THE PATTERN AND KEEP IT BILATERALLY SYMMETRIC? (Yes. Have a volunteer do so.)

Two examples of an extension of the above pattern are:

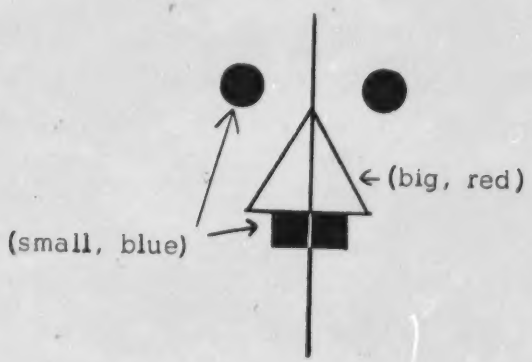



The children may continue making and adding to patterns singly or in small groups. The members of a group may work together or take turns. The "keep the symmetry" rule should be clearly understood -- the two sides of the pattern must be balanced.

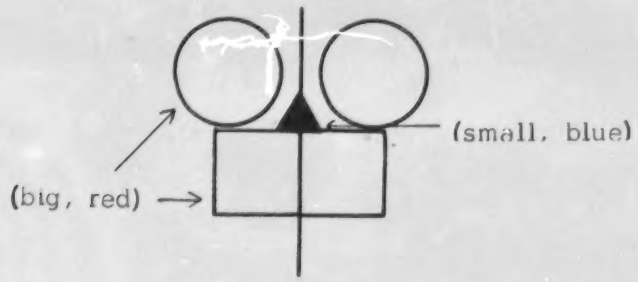
Activity B (Optional)

After a child or a group of children has made a bilaterally symmetric design, suggest that it be changed by following definite rules. You could give the first rule and have the children suggest later rules. In every case it should be noted that the resulting pattern is bilaterally symmetric.

For a rather complicated example assume property blocks have been used to form this design:



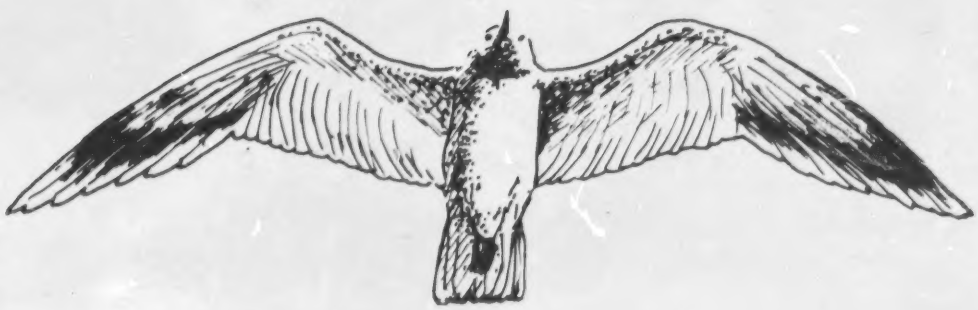
Then if the rule chosen for the transformation is red ⟷ blue and small ⟷ big the new design will be:




Some children can not handle a two-part rule such as this example and some others need to make the transformation in two separate steps.

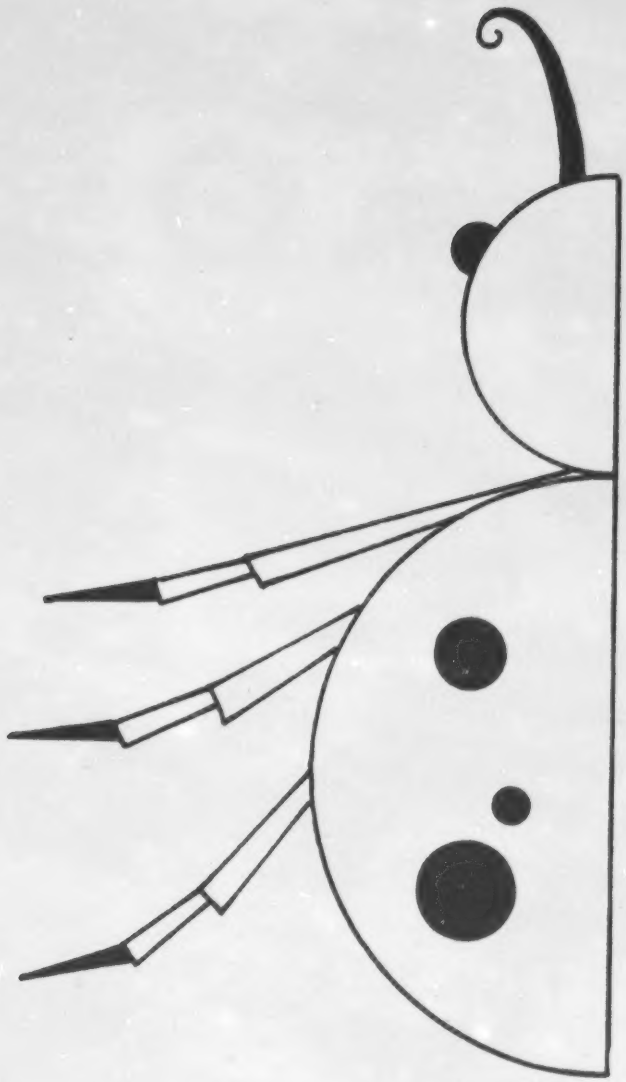
Activity C

Have the children complete the beetle on Design Sheet 3. They should understand that the completed beetle design should be bilaterally symmetric, but they should be assured that small variations are acceptable. Don't expect or urge perfect work.



Design Sheet 3
Unit 7

Complete the beetle.



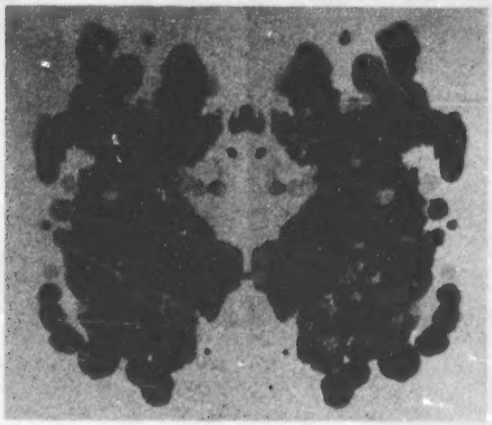
70

## ART ACTIVITIES FOR SECTION 3: BILATERAL SYMMETRY

Art activities that reinforce the concept of bilateral symmetry are suggested here. Any selection of them may be used any time after Lesson 8 has been completed.

Activity A: Paint-and-Fold Designs

Fold a sheet of rice paper or newsprint in half. Drip tempera paint on one half of the sheet, fold the other side over it and press together.



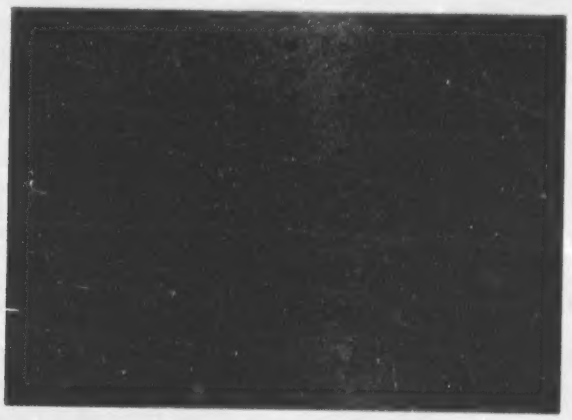
Activity B: Making Designs by Punching Holes

You will need colored construction paper and pencils or pointed sticks of various thicknesses.

1. Lay a sheet of paper over a peg board and with a pencil punch a small hole through the paper over each hole in the pegboard.

2. With pencils or pointed sticks of different thicknesses, the holes can be enlarged to make a pattern exhibiting bilateral symmetry.

3. Mount the punched sheet on another sheet of construction paper of contrasting color.



Variation

The children could also make their bilaterally symmetric patterns by laying the paper over the pegboard and punching just those holes they wish to have in their pattern.

Activity C: Rubbings

For this activity you will need newsprint, crayon or soft compressed charcoal, and objects with raised patterns.

1. Find objects with raised patterns exhibiting bilateral symmetry. Leaves and weeds are excellent for rubbings.

2. Place the newsprint over the object and rub the charcoal or crayon firmly over the paper.

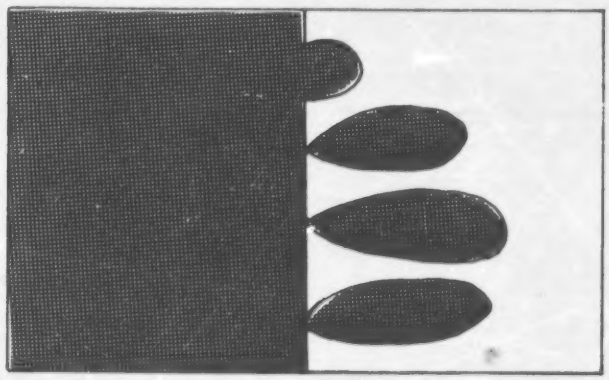
3. Mount the rubbings on a piece of construction paper for display.

The same technique can be used for other types of symmetry. A brick wall or tile floor give good examples of translational symmetry. A bracelet opened flat has a repeated pattern; a brooch may exhibit rotational symmetry.

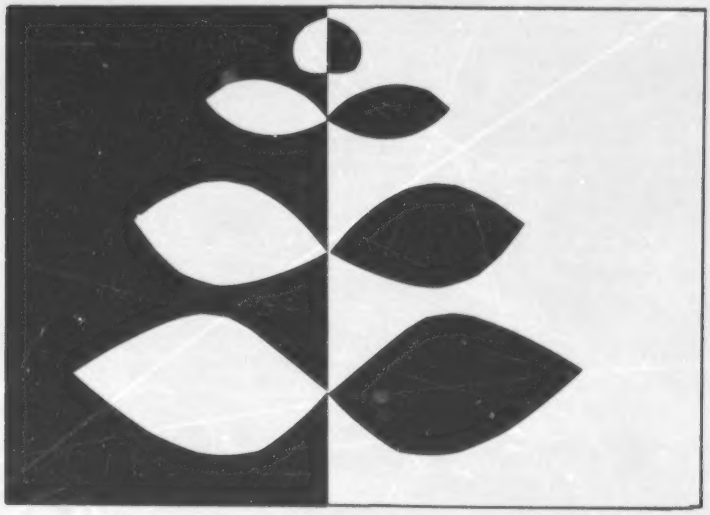
Activity D: Abstract Designs

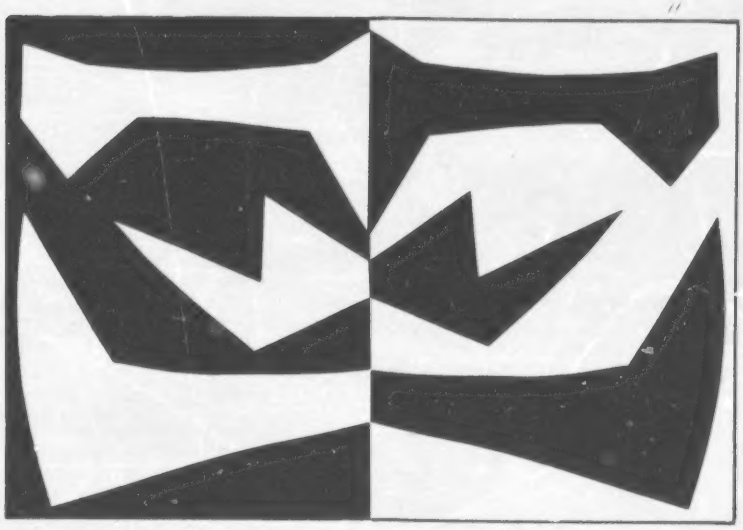
1. Have the children cut out 2 or 3 shapes from the 12" edge of a sheet of 9" x 12" colored construction paper. Make sure the cuts are on the 12" edge, not the 9" edge.

2. Put the shapes aside and paste the sheet down on one side of a 12" x 18" sheet of contrasting color.



3. Take the cut-out shapes and paste them down on paper on the opposite side of the center line in a flipped position. Good color combinations are blue paper on yellow, green on orange, or black on white. The patterns are bilaterally symmetric in shape but not in color.





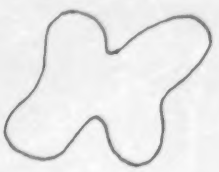
Activity E: Egg-Carton Animals

You will need tempera paint, construction paper, egg cartons, and pipe cleaners.

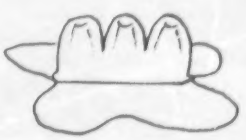
Butterfly

1. Cut out 3 humps from an egg carton.

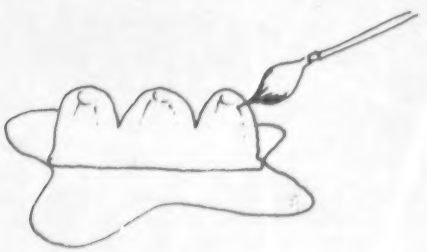
2. Cut out butterfly wings from construction paper.



3. Paste the egg carton segments onto the wings.



4. Paint the butterfly body and decorate wings.



5. Insert pipe cleaners for antennae.

Variations

The children may devise other ways of making animals or insects with the same materials. For example, a caterpillar can be made by cutting the whole egg carton longitudinally and then decorating the strip of 6 segments.

Activity F: Construction-Paper Figures

Children can make animal figures by first cutting out larger body parts (head, body, legs) from construction paper of various colors. Next, they paste these body parts together. Finally they cut out and add the other features (such as eyes, beak, feathers, etc.), carefully arranging them so as to retain bilateral symmetry.

Two-dimensional figure

1. To make a two-dimensional figure (e.g., an owl), cut out two circles, one larger than the other, for the head and body, and another piece representing a beak.

2. Paste these pieces down on a piece of construction paper.

3. Cut out two each of such features as eyes, feet, feathers, etc. These should be pasted on in a bilaterally symmetric pattern.

## SECTION 4 SYMMETRY IN SOUND AND MOVEMENT

Up to now, the children have been working exclusively with symmetries that can be observed visually. But their world is full of symmetries of other sorts. They walk symmetrically -- left, right, left, right. When they sing, they repeat rhythmic and melodic patterns. This section is designed to develop an awareness of repeating patterns of this type. The section is not divided into lessons, since opportunities for these activities vary greatly. We give a few samples for introducing these concepts, and expect that you will integrate lessons in symmetry with your music, physical education and language arts work, looking for symmetries wherever they may be found.

### RHYTHMIC MOVEMENT

Motor activities will help the children learn to recognize rhythmic patterns. Such physical education activities as the elephant walk are clearly symmetric. Other activities are listed below.

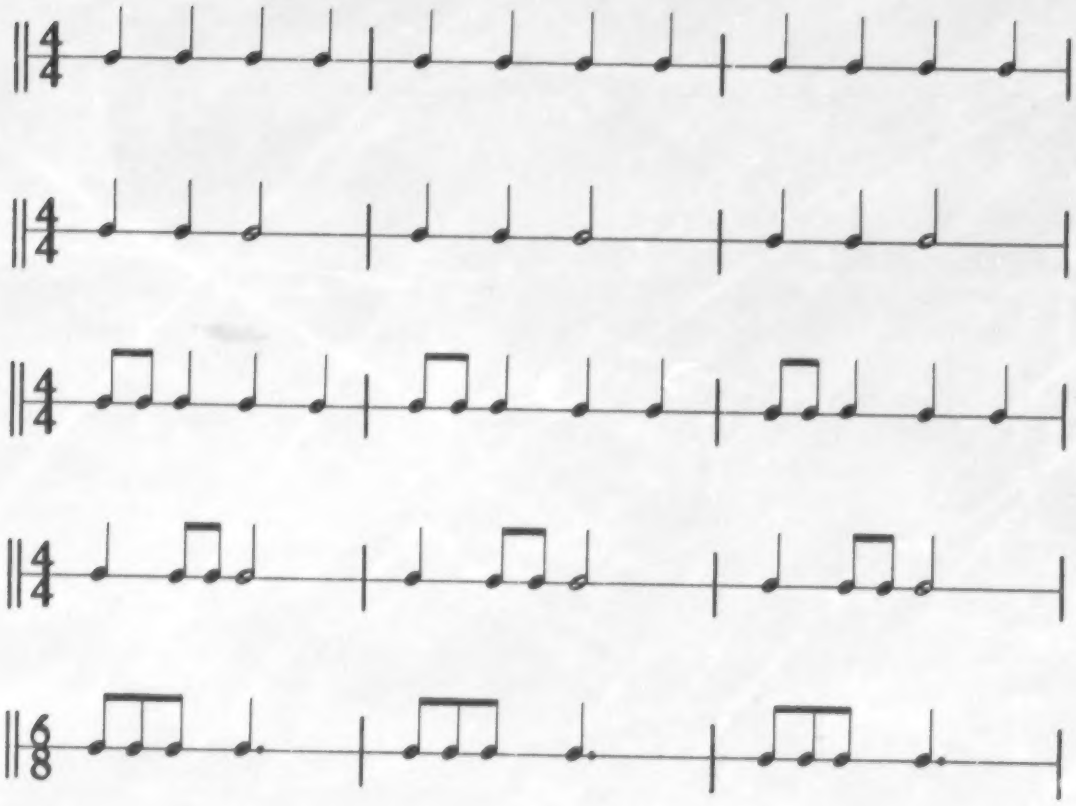
#### Choo-choo

This is a good start to linking physical rhythmic movement with sound. The children can form a long train, by each placing his hands on the waist of the child before him. They can shuffle around the room, with a "choo" for each step they take. When they are able to synchronize their feet with the "choos," they may be able to add a "toot" every second or third step.

#### Clapping and Stamping

The simplest rhythmic pattern the children can clap is a regular 1-2-3-4 with no variation in stress. When they have had sufficient practice clapping in unison, you might introduce stress by having them stamp their feet on each 1-beat. If the class has made oatmeal-box tom-toms, these might be used here. Rhythm patterns are suggested in order of increasing difficulty. (You may want to devise your own.) Illustrate each rhythm several times before asking the children to join you.

Introduce the activity by telling the children, "You have been looking at patterns of symmetry in many places. Now let us see if you can hear symmetric patterns. Listen first, and then see if you can clap the rhythm."



You might also have the children clap the rhythms of their favorite songs while they sing them or listen to records.

Rhythm Band

These instruments offer opportunity for greater variety than clapping. Some instruments might be used to beat the rhythm for the verses of a song while others beat the refrain. Some might be used just for accented beats. The variations will depend on the children's skill. A rhythm band can be used for marching, for accompanying records, or with class singing.

## MUSIC

The symmetry of repeated patterns is an essential element of music. In addition to the rhythmic patterns the children have been discovering in their work with rhythm instruments, music shows many other symmetric patterns. There are repetitions of single notes and of phrases, patterned melodic structures, and tonal patterns. Some of these are too complex for children to analyze; some may be recognized intuitively by a few children; some can be discovered easily. "Brother John" illustrates a number of symmetries within the grasp of a kindergarten class. You will note that the words and music follow the same repeated pattern.

If the children are able to sing it as a round, the form in which we have written it here should help illustrate the symmetric entry of one group of voices every two measures. The children, of course, would not be expected to read the musical notation, but they should find it easy to recognize visually the repeated pattern of the notes in each pair of phrases.

By indicating pitch level of "Are," "Brother" and "Morn" with your hand as they sing, you may help the children recognize the entry of each new phrase a third interval higher than the one before.

Other excellent examples of rounds and repeated patterns are attractively presented in Copy Cats by John LaMontaine, a book of very elementary piano pieces. If you can play the piano at all, you will have no difficulty with these.

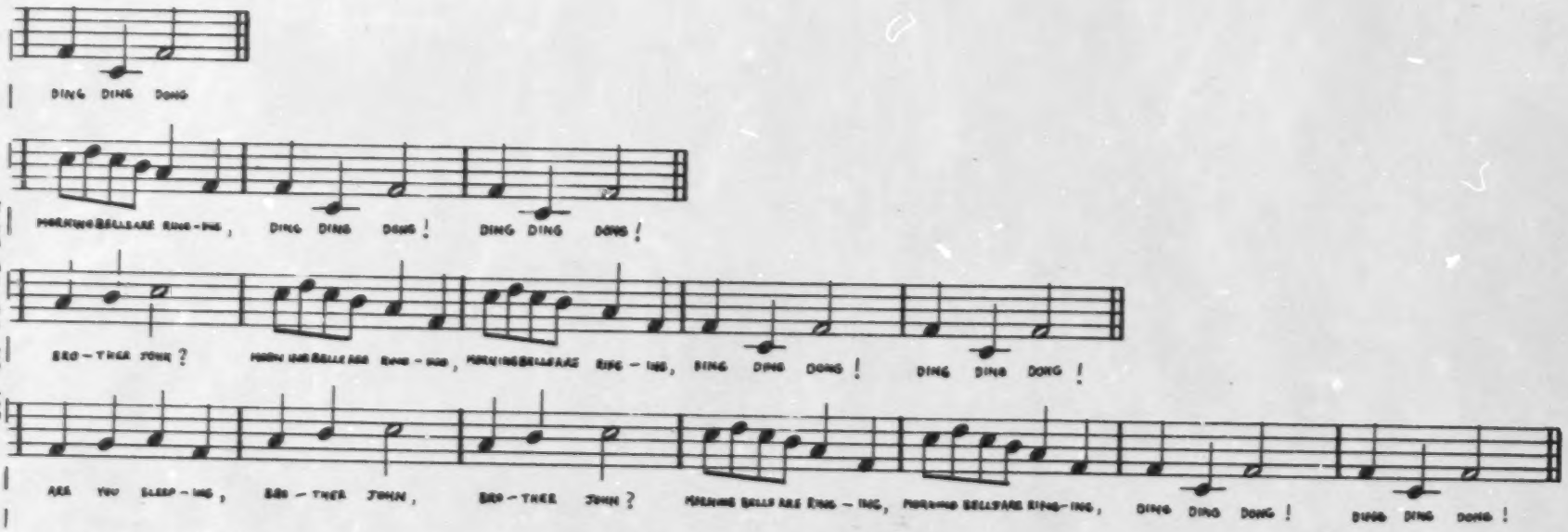
# BROTHER JOHN

ARE YOU SLEEP-ING, ARE YOU SLEEP-ING, BRO-THER JOHN, BRO-THER JOHN?
MORNING BELLS ARE RING-ING, MORNING BELLS ARE RING-ING, DING DING DONG!
ARE YOU SLEEP-ING, ARE YOU SLEEP-ING, BRO-THER JOHN, BRO-THER JOHN?
MORNING BELLS ARE RING-ING,
ARE YOU SLEEP-ING, ARE YOU SLEEP-ING, BRO-THER JOHN,
ARE YOU SLEEP-ING,

28

PASTE BOTTOM HALF HERE

CUT ON DOTTED LINE


DING DING DONG
MORNING BELLS ARE RING-ING, DING DING DONG! DING DING DONG!
BRO-THER JOHN? MORNING BELLS ARE RING-ING, MORNING BELLS ARE RING-ING, DING DING DONG! DING DING DONG!
ARE YOU SLEEP-ING, BRO-THER JOHN, BRO-THER JOHN? MORNING BELLS ARE RING-ING, MORNING BELLS ARE RING-ING, DING DING DONG! DING DING DONG!

CUT ON DOTTED LINE

## POETRY

The symmetries in poetry range from the most obvious to the most subtle. Rhythm and rhyme are of course the most easily recognized. There may also be repetition of words, of sounds, or of images. When you read a poem to the class and ask the children to raise their hands when they notice a repeated pattern, have each one identify the nature of the symmetry he has discovered.

In "Star Light" the children might note the fact that all the lines rhyme; they might count the four stressed syllables in each line.

Star light, star bright,
First star I see tonight,
I wish I may, I wish I might
Get the wish I wish tonight.

When you ask the children to write their own poems, it might be wise to give some examples with very simple rhythm and rhyme patterns.

I had a cat;
He caught a rat.
The rat was fat --
I hated that !

or

My father's car
Can ride so far,
I think that it
Can reach a star.